Quark-gluon hybrids (meson- and baryon-like) within a constituent quark-gluon model (CQGM)

Jorge Segovia

Physik-Department T30f
Technische Universität München

Nanjing University and Nanjing Normal University
May 2017
With the advent of the first particle accelerators, a large number of (lighter) hadrons were discovered in the 1950s and 1960s.

☞ Wolfgang Pauli: “Had I foreseen that, I would have gone into botany”.
☞ Enrico Fermi: “If I’d remember the names of these particles, I’d have been a botanist.”
The quark model (I)

☞ A classification scheme for hadrons in terms of their valence quarks and antiquarks:

☞ The quarks and antiquarks give rise the quantum numbers of the hadrons:

<table>
<thead>
<tr>
<th></th>
<th>d</th>
<th>u</th>
<th>s</th>
<th>c</th>
<th>b</th>
<th>t</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q - Electric charge</td>
<td>-1/3</td>
<td>+2/3</td>
<td>-1/3</td>
<td>+2/3</td>
<td>-1/3</td>
<td>+2/3</td>
</tr>
<tr>
<td>I - Isospin</td>
<td>+1/2</td>
<td>+1/2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>I_z - Isospin z-component</td>
<td>-1/2</td>
<td>+1/2</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>S – strangeness</td>
<td>0</td>
<td>0</td>
<td>-1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>C – charm</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>+1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>B – Bottomness</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-1</td>
<td>0</td>
</tr>
<tr>
<td>T – Topness</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>+1</td>
</tr>
</tbody>
</table>

☞ Underlies “flavor SU(3)” symmetry

Murray Gell-Mann
George Zweig
Successful classification scheme organizing the large number of conventional hadrons

Baryons

Mesons

$$3 \otimes 3 \otimes 3 = 10_S \oplus 8_M \oplus 8_M \oplus 1_A$$

$$3 \otimes \bar{3} = 8 \oplus 1$$

What the Particle Data Group is...

AUTHORS of the 2016 RPP edition

The Particle Data Group is an international collaboration charged with summarizing Particle Physics, as well as related areas of Cosmology and Astrophysics. In 2014, the PDG consists of 205 authors from 140 institutions in 24 countries.

The Review of Particle Physics includes a compilation and evaluation of measurements of the properties of the elementary particles including gauge bosons, Higgs bosons, leptons, quarks, mesons, and baryons. It summarizes searches for hypothetical particles such as heavy neutrinos, supersymmetric and technicolor particles, axions, dark photons, etc.

There also 112 review articles on topics such as Higgs bosons, supersymmetry, Big Bang nucleosynthesis, probability, statistics, accelerators and detectors.

The summaries are published in even-numbered years as a new 1675-page book, the Review of Particle Physics, and as an abbreviated version (317 pages), the Particle Physics Booklet. The Review is published in a major journal, and in addition the PDG distributes 15,000 copies of the book and 31,000 copies of the booklet. The Review has been called the bible of particle physics; over the years, it has been cited in more than 50,000 papers.

In the 2014 Review, the listings include 3,283 new measurements from 899 papers, in addition to 32,000 measurements from 9,000 papers that appeared in earlier editions. Evaluations of these properties are abstracted in summary tables.

All tables, listings, and reviews (and errata) are also available on the Particle Data Group website: http://pdg.lbl.gov.
Charmonium and bottomonium states were discovered in the 1970s. Experimentally clear spectrum of narrow states below the open-flavor threshold.

Heavy quarkonia are bound states made of a heavy quark and its antiquark (\(c\bar{c}\) charmonium and \(b\bar{b}\) bottomonium).

They can be classified in terms of the quantum numbers of a nonrelativistic bound state \(\rightarrow\) Reminds positronium [(\(e^+e^-\))-bound state] in QED.

Heavy quarkonium is a very well established multiscale system which can serve as an ideal laboratory for testing all regimes of QCD.
The discovery of the X(3872)

- In 2003, Belle observed an unexpected enhancement in the $\pi^+\pi^- J/\psi$ invariant mass spectrum while studying $B^+ \rightarrow K^+\pi^+\pi^- J/\psi$.

- It was later confirmed by BaBar in B-decays and by both CDF and D0 at Tevatron in prompt production from $p\bar{p}$ collisions.

- Its quantum numbers, mass, and decay patterns make it an unlikely conventional charmonium candidate.
Many experiments around the world

The scientific community has witnessed an explosion of related experimental activity

BELLE@KEK (Japan)

BABAR@SLAC (USA)

CLEO@CORNELL (USA)

PANDA@GSI (Germany)

Jorge Segovia (jorge.segovia@tum.de)

Quark-gluon hybrids within a constituent quark-gluon model
The XYZ particles – A new particle zoo?

Jorge Segovia (jorge.segovia@tum.de)

- Belle observed $Z(4430)^-$ from sample of $\sim 2$ k $B^0 \rightarrow \psi(2S) K^{+,-,0} \pi^+$
- Charged state $\Rightarrow$ minimal quark content of $c\bar{c}u\bar{d}$


Belle: 1D fit to $m(\psi(2S)\pi^-)$

\[ M = 4433 \pm 4 \pm 2 \text{ MeV/c}^2 \]
\[ \Gamma = 45^{+18}_{-13} \pm 30 \text{ MeV/c}^2 \]

- $Y(4360)$ - confirmed for the first time with much better resonance parameters.
- $Y(4660)$ - A 5.8$\sigma$ narrow state discovered.

Two solutions: constructive and destructive interference.

- $M(\psi(3630)) = 3943 \pm 11 \pm 13 \text{ MeV}$
- $\Gamma_{tot} = 87 \pm 22 \pm 26 \text{ MeV}$

- $M(\psi(3934)) = 3942 \pm 7 \pm 6 \text{ MeV}$
- $\Gamma_{tot} = 37^{+26}_{-15} \pm 12 \text{ MeV}$

693/ fb, PRL 100, 202001

253/ fb, PRL 94, 182002

395/ fb, PRL 96, 082003
Summary of the XYZ particles


Similar structures have been discovered in other quark sectors such as the $J^{PC} = 1^{-+}$ candidates $\pi_1(1400)$ \cite{PLB_657_2007_27-31} and $\pi_1(1600)$ \cite{PRL_104_2010_241803}
The states that do not fit into the quark model are called Exotics
(Keep in mind that QCD's spectrum will inevitably be richer than baryons and mesons)

☞ Glueball (only gluons)
An hypothetical composite particle which consists solely of gluon particles, without valence quarks.

☞ Hybrid (Q̅Qg)
Exotic properties are due to gluonic excitations.

☞ Molecule (Q̅q − ̅Qq)
Shallow bound states of heavy mesons analogous to the deuteron.

☞ Diquarkonium (Qq − ̅Q̅q)
The constituent quarks are assumed to be clustered into color triplet diquarks.

☞ Hadroquarkonium (Q̅Q − q̅q)
A compact core that is a color-singlet Q̅Q surrounded by light mesons.

And so on...
QCD’s Key feature

Quantum Electrodynamics (QED)
- Theory of the electroweak interaction.
- d.o.f.: electrons and photons.
- No Photon self-interactions.

Quantum Chromodynamics (QCD)
- Theory of the strong interaction.
- d.o.f.: quarks and gluons.
- **GLUON SELF-INTERACTIONS.**

Origin of confinement, DCSB, ... → How does glue manifest itself in low energy regime?

- Possible clues looking at hadrons with explicit gluonic d.o.f.
  *Same role played by gluons and quarks in making matter!!*

- Hybrid mesons with a heavy-quark pair are the most amenable to theoretical treatment.

- **LHCb@CERN, GlueX@JLab12 and PANDA@FAIR are producing a rich environment of gluons in order to promote the formation of glueballs and quark-gluon hybrids.**
Theoretical study of glueballs and hybrids

☞ **Glueballs.**

- It is difficult to single out which states of the hadronic spectrum are glueballs because we lack the necessary knowledge to determine their decay properties.

- The strong expected mixing between glueballs and conventional quarkonia leads to a broadening of the possible glueball states, not simplifying their isolation.

☞ **Hybrids.**

- Valence gluonic degrees of freedom increase the quantum numbers that are available to fermion-antifermion systems.

- Some of them cannot be confused with ordinary quark-antiquark mesons and, moreover, they do not mix with conventional quark model states.

- At lowest order, hybrids decay into a pair of mesons with a valence gluon decaying into a quark-antiquark pair followed by a color re-arrangement process.

☞ **Two broad ideas concerning soft glue:**

- A local quasi-particle degree of freedom.
  - (MIT) bag model.
  - Quasi-gluon model.
  - Constituent gluon model.

- Collective non-local degrees of freedom.
  - Flux-tube model.
  - EFT description and Born-Oppenheimer approximation.
  - Hybrid static energies in Lattice QCD.
The quarks in the interior of a bag have small (current) masses and feel only weak forces whereas in the exterior the quarks are not allowed to propagate.

A gluonic field is also placed in the interior of a bag with appropriate boundary conditions. This gives \((1^{+-}, TM)\) or \((1^{+-}, TE)\) solutions, with TE modes the lightest.


General relevant features:

- The bag energy density \(B\) is taken to be universal because it is a property of the complex structure of the QCD vacuum exterior to the hadronic bag.
- The energy is given by
  \[
  E = \frac{4}{3} \pi R^3 B + \sum_{\text{constituents}} \frac{E_{\text{mode}}}{R} + \frac{Z_0}{R} + \Delta E(\alpha_s^n),
  \]
- Some difficulties are: (i) computation of gluon self-energies, (ii) existence of spurious degrees of freedom associated with the center of mass, and (iii) determination of the bag’s response when quarks and gluons are present.

Consequences in the meson sector:

- The lowest-lying hybrid multiplet \(J^{PC} = 1^{+-}, (0, 1, 2)^{--}\) is constructed from a \(q\bar{q}\) color octet with \(J^{PC} = 0^{++}\) or \(1^{--}\) and a TE gluon with \(J^{PC} = 1^{+-}\).
- Spin splittings follow the pattern observed by lattice QCD which is due to the interaction of the valence gluon with the valence (anti-)quark through a TE field.
The massless gluon is one more constituent besides quarks and antiquarks meaning that hybrid mesons and baryons are, respectively, 3- and 4-body bound state systems.

*D. Horn and J. Mandula, Phys. Rev. D17 (1978) 898*

**Consequences in the meson sector:**
- The effective Hamiltonian:

\[
H_{\text{eff.}} = m_q + m_{\bar{q}} + \frac{\vec{p}_q^2}{2m_q} + \frac{\vec{p}_{\bar{q}}^2}{2m_{\bar{q}}} + |\vec{p}_g| + G(|\vec{r}_q - \vec{r}_g| + |\vec{r}_{\bar{q}} - \vec{r}_g|) + V_0
\]

- The heavy quarks are treated non-relativistically whereas the gluon is massless and thus its kinetic energy is the absolute value of its momentum.
- The interaction between the quark (antiquark) and the gluon is represented by an attractive linear potential.
- A weak repulsive quark-antiquark force is lumped with other short-range effects into the undetermined constant.

| $|K|$ | $\eta$ | $\xi$ | $l$ | $l_{q\bar{q}}$ | $l_g$ | $s_{q\bar{q}} = 0$; $J^{PC}$ | $s_{q\bar{q}} = 1$; $J^{PC}$ |
|-----|------|------|---|--------|------|------------|------------|
| 0   | +    | +    | 0 | 0      | 0    | $1^{+-}$    | (0, 1, 2)$^{++}$ |
| 0   | +    | +    | 1 | 1      | 0    | $0 (0, 1, 2)^{-+}$ | (0, 1, 1, 2, 3)$^{--}$ |
| 1   | −    | +    | 1 | 0      | 1    | $0 (0, 1, 2)^{-+}$ | (0, 1, 1, 2, 3)$^{--}$ |
| 0   | +    | −    | 1 | 0      | 1    | $0 (0, 1, 2)^{-+}$ | (0, 1, 1, 2, 3)$^{--}$ |

- The constituent gluon has $J^{PC}|_g = 1^{--}$ quantum numbers instead of the $J^{PC}|_g = 1^{+-}$ for the gluon field:

\[
J^{PC}|_g = 1^{--} \quad \{1^{+-}; (0, 1, 2)^{++}\} \longrightarrow J^{PC}|_g = 1^{+-} \quad \{1^{--}; (0, 1, 2)^{-+}\}
\]
Consequences in the meson sector:

- The Schrödinger equation for the radial wave function (in momentum space):

  \[
  \left[ 2m + \frac{q^2}{m} + \frac{k^2}{4m} + \Sigma_g(k) + \Sigma_q \right] \Psi_\alpha(k, q) - \frac{1}{2N_C} \sum_{\alpha'} \int \frac{q'^2 dq'}{(2\pi)^3} V_{Q\bar{Q}}(q, \alpha; q', \alpha') \Psi_\alpha(k, q') \\
  + \sum_{\alpha'} \int \frac{k'^2 dk'}{(2\pi)^3} \frac{q'^2 dq'}{(2\pi)^3} V_{Q\bar{Q}g}(k, q, \alpha; k', q', \alpha') \Psi_\alpha(k', q') = M \Psi_\alpha(k, q)
  \]

- Kinetic and self-energy terms for the quarks and gluons.
- The quark-antiquark potential in color octet.
- The attractive (anti-)quark-gluon interactions.
- Irreducible three-body interaction.

In the quasi-particle approximation, the gluelumps are the constituent gluons and they should be coupled to the \(Q\bar{Q}\)-pair.

The irreducible 3-body interaction is responsible for producing the inverted parity ordering of the gluelump spectra and thus of the \(Q\bar{Q}g\) quarkonium spectra.

The two lowest gluelumps have quantum numbers \(1^{++}\) and \(1^{--}\) that coupled with \(Q\bar{Q}\) in an S-wave: \(\{1^{--}, 0^{-+}, 1^{++}, 2^{--}\}\) and \(\{1^{-+}, 0^{++}, 1^{+-}, 2^{++}\}\).
Flux-tube model for hybrid mesons

Hamiltonian approach in which the degrees of freedom are quarks (antiquarks) connected by gluons that are condensed into collective string-like flux-tubes


Consequences in the meson sector:

- The lowest excitations of the string will correspond to nonrelativistic, small, transverse displacements oscillations:

\[ H_S = b_0 r + \frac{1}{b_0 r} \sum_m \sum_{i=1}^2 \left( \pi_m^i \pi_m^i + \frac{1}{4} \omega_m^2 b_0^2 r^2 q_m^i q_m^i \right) \]

- Quantization of the equation above:

\[ E_0 - b_0 r = \sum_{m=1}^{N} \sum_{i=1}^{2} \frac{1}{2} \omega_m = \frac{\sqrt{2}}{a} \left[ \frac{\sin(\pi N/4(N + 1))}{\sin(\pi/4(N + 1))} \right] \approx \left( \frac{4}{\pi a^2} \right) r - \frac{1}{a} - \frac{\pi}{12r} + \ldots \]

- The eigenenergy of the string eigenstate will trace out an adiabatic potential, \( E^S(r) \), that describes the physics of a hybrid meson:

\[ H^{(1)} = -\frac{1}{2\mu} \frac{\partial^2}{\partial r^2} + \frac{l(l + 1) - \Lambda^2 + \langle L_s^2 \rangle}{2\mu r^2} + E^{(1)}(r) \]

\[ E^{(1)}(r) = -\frac{4\alpha_s}{3r} + c + br + \frac{\pi}{r} \left( 1 - e^{-fb^{1/2}r} \right) \]

Hybrid mesons are constructed by specifying the gluonic states via phonon operators and combining these with quark operators. The lowest multiplet predicted is \( 1^{\pm\pm} \), \( (0, 1, 2)^{\pm\mp} \), in contradiction with lattice QCD computations.
EFT description for hybrid mesons

**Heavy quarkonium hybrids** are characterized by the vast dynamical difference between the slow and massive quarks and the fast and massless gluons


Consequences in the meson sector:

- The Born-Oppenheimer approximation can be employed to replace the fast gluon field by an effective potential between the nonrelativistic quarks.
- The effective potentials originate from excited gluon configurations and can be reliably computed using lattice gauge simulations (next slide).
- The motion of the $Q$ and $\bar{Q}$ can be described by the Schrödinger equation with potential $V_\Gamma(r)$.

\[
\left[ -\frac{1}{mr^2} \partial_r r^2 \partial_r + \frac{1}{mr^2} \begin{pmatrix} l(l+1) + 2 & 2\sqrt{l(l+1)} & 0 \\ 2\sqrt{l(l+1)} & l(l+1) & 0 \\ 0 & 0 & l(l+1) \end{pmatrix} + \begin{pmatrix} V_\Sigma(r) \\ V_{-\pi} \\ V_{+\pi} \end{pmatrix} \right] \Psi = E \Psi
\]

- The eigenstates of the gluonic system must be organized in representations of the cylindrical symmetry group $D_{\infty h}$. 

Jorge Segovia (jorge.segovia@tum.de)
They are Nonperturbative quantities!

$E_{\Sigma^+}(r)$ is the ground state potential that generates the standard Quarkonium states.

The rest of the static energies correspond to gluonic excitations that generate hybrids.

The two lowest hybrid static energies are $E_{\Pi_u}^{(0)}(r)$ and $E_{\Sigma_u^-}^{(0)}(r)$.

$\rightarrow$ Nearly degenerate at short distances.

Good agreement was found between quenched and unquenched computations.

K.J. Juge et al. PRL 90 (2003) 161601
Why do we want to construct a phenomenological model?

The construction of a reliable model of hybrid meson structure and dynamics is important for the interpretation of experimental results.

Among its advantages:
- Solving the 3- and 4-body bound state problem is quite standard nowadays.
- The quark-quark and quark-gluon interactions are known. The three-body interaction can be discussed.
- Particular decay models for hybrids have been proposed and their computational method is quite similar to the one presented for the $^3P_0$ decay model.
- Other hadron properties like electromagnetic properties are easy to characterized as soon as hybrid wave functions are computed.

Against Lattice computations
- It is expensive to compute large numbers of experimentally relevant quantities on the lattice.
- It is also likely that the computation of complicated amplitudes involving hybrids will remain out of reach of lattice methods for a long time.

Against EFT computations
- It is mostly based on the fact that valence quarks (antiquarks) are heavy and thus one can separate gluonic degrees of freedom from quark dynamics.
- They rely on the computation of hybrid static energies in Lattice QCD making this approach dependent on the one above.
- It is not yet clear how to deal with spin-dependent corrections, mixture of different adiabatic surfaces, and how hybrids should decay.
Scattering case:

\[
V(r) = C_1 \alpha_s \left\{ \frac{1}{r} + \pi \left( \frac{6 - 4S_+^2}{3m_1m_2} - \frac{1}{2m_1^2} - \frac{1}{2m_2^2} \right) \delta^3(r) - \frac{1}{2m_1m_2} \left[ \frac{p_1 \cdot p_2}{r} + \frac{(p_1 \cdot r)(p_2 \cdot r)}{r^3} \right] \right. \\
- \left( \frac{m_2^2 + m_1^2 + 4m_1m_2}{4m_1^2m_2^2} \right) \frac{L \cdot S_+}{r^3} + \frac{1}{4m_1m_2} \frac{S_- \cdot \Omega}{r^3} + \left( \frac{m_2^2 - m_1^2}{4m_1m_2} \right) \frac{L \cdot S_-}{r^3} \\
+ \frac{(m_2 - m_1)}{4(m_1 + m_2)m_1m_2} \frac{S_+ \cdot \Omega}{r^3} + \frac{1}{2m_1m_2} \left[ \frac{S_+^2}{r^3} - 3 \left( S_+ \cdot \frac{r}{r^5} \right)^2 \right] \right\}
\]

Annihilation case:

\[
V(r) = C_4 \frac{2\pi \alpha_s}{m^2} \left[ \frac{3}{4} + S_1 \cdot S_2 \right] \delta^3(r) = C_4 \frac{\pi \alpha_s}{m^2} S_+^2 \delta^3(r)
\]

Short-range potentials of QCD (II)

\[ V_{\text{sca}}(r) = C_7 \alpha_s \left\{ \frac{1}{r} - \frac{1}{2m_g m_q} \left[ \frac{p_g \cdot p_q}{r} + \frac{(p_g \cdot r)(p_q \cdot r)}{r^3} \right] \right. \\
+ \pi \left( \frac{11}{3m_g m_q} - \frac{2}{3m_q^2} - \frac{1}{2m_q} - \frac{4}{3m_g m_q} S^2_+ \right) \delta^3(r) - \left( \frac{m_q^2 + m_g^2 + 4m_g m_q}{4m_g^2 m_q^2} \right) \frac{L \cdot S_+}{r^3} \right. \\
+ \frac{1}{4m_g m_q} \frac{S^- \cdot \Omega}{r^3} + \frac{(m_q - m_g)}{4m_q m_g (m_q + m_g)} \frac{S_+ \cdot \Omega}{r^3} + \left( \frac{m_g^2 - m_q^2}{4m_q^2 m_g^2} \right) \frac{L \cdot S_-}{r^3} \right. \\
\left. + \frac{1}{2m_g m_q} \left[ \frac{S^2_+}{r^3} - 3 \left( \frac{S_+ \cdot r}{r^5} \right)^2 \right] + \frac{m_q - m_g}{m_g^2 m_q} Q(r) \right\} \\

\[ V_{\text{com}}(r) = \frac{2\pi C_8 \alpha_s}{m_g (m_g + 2m_q)} \left[ \frac{15}{4} - S^2_+ \right] \delta^3(r) + C_{10} \alpha_s \frac{m_q}{2m_g} \left[ S^2_+ - \frac{7}{4} \right] \frac{e^{-m_q r}}{r} \]

Linear screened potential:

\[ V_{\text{CON}}(r_{ij}) = \left[ -a_c(1 - e^{-\mu_c r_{ij}}) + \Delta \right] (\lambda_i \cdot \lambda_j) \]

- \( r_{ij} \to 0 \quad \Rightarrow \quad V_{\text{CON}}(r_{ij}) \to (-a_c\mu_c r_{ij} + \Delta)(\lambda_i \cdot \lambda_j) \quad \Rightarrow \quad \text{Linear.} \)
- \( r_{ij} \to \infty \quad \Rightarrow \quad V_{\text{CON}}(r_{ij}) \to (-a_c + \Delta)(\lambda_i \cdot \lambda_j) \quad \Rightarrow \quad \text{Threshold.} \)
Same central parts for the scalar and vector Lorentz structures of the confinement:

\[ V^{C,\text{scalar}}_{\text{CON}}(r_{ij}) = V^{C,\text{vector}}_{\text{CON}}(r_{ij}) = \left[-a_c(1 - e^{-\mu_c r_{ij}}) + \Delta\right](\lambda_i^c \cdot \lambda_j^c) \]

There are different spin-dependent corrections related with the scalar or vector Lorentz character of the confinement

\[ V^{SS}_{\text{CON}}(r_{ij}) = \frac{1}{6m_i m_j}(\sigma_i \cdot \sigma_j) \nabla^2 V^C_{\text{CON}}(r_{ij}) \]

\[ V^{T}_{\text{CON}}(r_{ij}) = \frac{1}{12m_i m_j} \left( \frac{1}{r} \frac{dV^C_{\text{CON}}(r_{ij})}{dr_{ij}} - \frac{d^2 V^C_{\text{CON}}(r_{ij})}{dr_{ij}^2} \right) S_{ij} \]

\[ V^{SO}_{\text{CON}}(r_{ij}) = \frac{1}{4m_i^2 m_j^2} \frac{1}{r} \frac{dV^C_{\text{CON}}(r_{ij})}{dr_{ij}} \left[ ((m_i + m_j)^2 + 2m_i m_j)(S_+ \cdot L) + (m_j^2 - m_i^2)(S_- \cdot L) \right] \]

The final expressions are:

\[ V^{SO}_{\text{CON}}(r_{ij}) = -\left(\lambda_i^c \cdot \lambda_j^c\right) \frac{a_c \mu_c e^{-\mu_c r_{ij}}}{4m_i^2 m_j^2 r_{ij}} \left[ ((m_i^2 + m_j^2)(1 - 2a_s)
+ 4m_i m_j(1 - a_s))(S_+ \cdot L) + (m_j^2 - m_i^2)(1 - 2a_s)(S_- \cdot L) \right] \]

\[ V^{T}_{\text{CON}}(r_{ij}) = -\left(\lambda_i^c \cdot \lambda_j^c\right) \frac{a_c \mu_c e^{-\mu_c r_{ij}}}{12m_i m_j r_{ij}}(1 - a_s)(1 + \mu_c r_{ij})S_{ij} \]

\[ V^{SS}_{\text{CON}}(r_{ij}) = -\left(\lambda_i^c \cdot \lambda_j^c\right) \frac{a_c \mu_c e^{-\mu_c r_{ij}}}{6m_i m_j r_{ij}}(1 - a_s)(2 - \mu_c r_{ij})(\sigma_i \cdot \sigma_j) \]

J. Segovia, PhD thesis
Light quark sector: Goldstone-boson exchange potentials

QCD Lagrangian invariant under the chiral transformation

\[ \mathcal{L} = \bar{\psi} \left( i \gamma^\mu \partial_\mu - M(q^2) U^{\gamma_5} \right) \psi \]

Chiral symmetry is spontaneously broken

\[ U^{\gamma_5} = \exp \left( i \pi^a \lambda^a \gamma_5 / f_\pi \right) \]

\[ \sim 1 + \frac{i}{f_\pi} \gamma^5 \Lambda^a \pi^a - \frac{1}{2f_\pi^2} \pi^a \pi^a + \ldots \]

Pseudo-Goldstone Bosons (\( \pi \), \( K_i \) and \( \eta_8 \))

Constituent quark mass

\[ M(q^2) = m_q F(q^2) = m_q \left[ \frac{\Lambda^2}{\Lambda^2 + q^2} \right]^{1/2} \]

Rapid acquisition of mass is effect of gluon cloud


J. Segovia, PhD thesis
Gaussian Expansion Method (GEM). Two-body bound state problem (I)

Two-body Schrödinger equation:
\[
\left[ -\frac{1}{2\mu} \nabla^2 + V(r) - E \right] \psi_{lm}(r) = 0
\]

Expand \( \psi_{lm}(r) \) in terms of a set of Gaussian basis functions:
\[
\psi_{lm}(r) = \sum_{n=1}^{n_{\text{max}}} c_{nl} \phi_{nlm}^{G}(r), \quad \phi_{nlm}^{G}(r) = \phi_{nl}^{G}(r) Y_{lm}(\hat{r})
\]

where
\[
\phi_{nl}^{G}(r) = N_{nl} r^l e^{-\nu_n r^2}, \quad N_{nl} = \left( \frac{2^{l+2} (2\nu_n)^{l+\frac{3}{2}}}{\sqrt{\pi} (2l + 1)!!} \right)^{\frac{1}{2}}
\]

The best set of Gaussian size parameters are those in geometric progression:
\[
\nu_n = \frac{1}{r_n^2}, \quad r_n = r_1 a^{n-1} \quad (n = 1, \ldots, n_{\text{max}})
\]

Dense distribution in the short-range region \( \Rightarrow \) Short-range correlations
Coherent superposition in the asymptotic region \( \Rightarrow \) Exponentially-damped tails

Rayleigh-Ritz variational principle and a generalize matrix eigenvalue problem:
\[
\sum_{n'=1}^{n_{\text{max}}} \left[ (T_{nn'} + V_{nn'}) - EN_{nn'} \right] c_{n'l} = 0 \quad (n = 1, \ldots, n_{\text{max}})
\]
Some examples of matrix elements are:

\[ N_{nn'} = \left( \frac{2 \sqrt{\nu_n \nu_{n'}}}{\nu_n + \nu_{n'}} \right)^{l+\frac{3}{2}} \]

\[ T_{nn'} = \langle \phi_{nlm}^G | - \frac{1}{2\mu} \nabla^2 | \phi_{n'lm}^G \rangle = \frac{1}{\mu} \left( \frac{2l + 3}{\nu_n + \nu_{n'}} \right)^{\frac{1}{2}} \left( \frac{2 \sqrt{\nu_n \nu_{n'}}}{\nu_n + \nu_{n'}} \right)^{l+\frac{3}{2}} \]

\[ V_{nn'} = \phi_{nlm}^G | V(r) | \phi_{n'lm}^G \rangle = N_{nl} N_{n'l} \int_0^\infty r^{2l} e^{-(\nu_n + \nu_{n'}) r^2} V(r) r^2 dr \]

For three explicit forms of \( V(r) \):

\[ \langle \phi_{nlm}^G | r^2 | \phi_{n'lm}^G \rangle = \frac{l + \frac{3}{2}}{\nu_n + \nu_{n'}} \left( \frac{2 \sqrt{\nu_n \nu_{n'}}}{\nu_n + \nu_{n'}} \right)^{l+\frac{3}{2}} \]

\[ \langle \phi_{nlm}^G | \frac{1}{r} | \phi_{n'lm}^G \rangle = \frac{2}{\sqrt{\pi}} \frac{2^l l!}{(2l + 1)!!} \sqrt{\nu_n + \nu_{n'}} \left( \frac{2 \sqrt{\nu_n \nu_{n'}}}{\nu_n + \nu_{n'}} \right)^{l+\frac{3}{2}} \]

\[ \langle \phi_{nlm}^G | e^{-\mu r^2} | \phi_{n'lm}^G \rangle = \left( \frac{2 \sqrt{\nu_n \nu_{n'}}}{\nu_n + \nu_{n'} + \mu} \right)^{l+\frac{3}{2}} \]

The generalization for coupled-channels can be read as

\[
\sum_{n'=1}^{n_{\text{max}}} \left[ (T_{nn'}^\alpha - E N_{nn'}^\alpha) c_{n'l}^\alpha + \sum_{\alpha'} \sum_{c_{n'l}^\alpha'} V_{nn'}^{\alpha\alpha'} c_{n'l}^\alpha' \right] = 0
\]

where \( T_{nn'}^\alpha \) and \( N_{nn'}^\alpha \) are diagonal and the mixing is given by \( V_{nn'}^{\alpha\alpha'} \).
The complex-range Gaussian expansion method is introduced in order to reproduce highly oscillatory functions.

- Such oscillating functions can appear in highly excited vibrational states of few-body systems.
- Eigenfunctions of a harmonic oscillator Hamiltonian potential? \( \rightarrow \) They are not useful because the calculation of the matrix elements with them is very hard.

The complex-range Gaussian basis functions are given by:

\[
\phi_{nl}^{GC} = N_{nl}^{GC} r^l e^{-\nu_n r^2} \cos(\alpha \nu_n r), \\
\phi_{nl}^{GS} = N_{nl}^{GS} r^l e^{-\nu_n r^2} \sin(\alpha \nu_n r),
\]

The parameter \( \alpha \) is a free parameter in principle, but numerical tests suggest \( \alpha \sim \pi/2 \).

The reason why the functions \( \phi_{nl}^{GC} \) and \( \phi_{nl}^{GS} \) are easy to be used in numerical calculations is as follows:

\[
\phi_{nl}^{GC} = N_{nl}^{GC} r^l \frac{e^{-\eta_n r^2} + e^{-\eta_n^* r^2}}{2}, \\
\phi_{nl}^{GS} = N_{nl}^{GS} r^l \frac{e^{-\eta_n r^2} + e^{-\eta_n^* r^2}}{2i}
\]

with complex size parameters:

\[
\eta_n = (1 + i\alpha)\nu_n, \quad \eta_n^* = (1 - i\alpha)\nu_n
\]
The 3-body Schrödinger equation is given by (for the case of central forces alone):
\[
T + V^{(1)}(r_1) + V^{(2)}(r_2) + V^{(3)}(r_3) - E \]
\[
\Psi_{JM} = 0
\]

The wave function is described as a sum of amplitudes of 3 rearrangement channels
\[
\Psi_{JM} = \Phi_{JM}^{(i=1)}(r_1, R_1) + \Phi_{JM}^{(i=2)}(r_2, R_2) + \Phi_{JM}^{(i=3)}(r_3, R_3)
\]

Each amplitude is expanded in terms of the Gaussian basis functions written in Jacobian coordinates \(r_i\) and \(R_i\):
\[
\Phi_{JM}^{(i)}(r_i, R_i) = \sum_{n_i,l_i,m_i, N_i,L_i,M_i} A_{n_i,l_i,m_i,N_i,L_i,M_i}^{(i)} \left[ \phi_{n_i,l_i,m_i}^G(r_i) \psi_{N_i,L_i,M_i}^G(R_i) \right]_{JM} \quad (i = 1 - 3)
\]

- The \(l_i\) and \(L_i\) are restricted to \(0 \leq l_i \leq l_{\text{max}}\) and \(|J - l_i| \leq L_i \leq J + l_i\).
- Eigenenergy and coefficients \(\rightarrow\) Rayleigh-Ritz variational principle.
Given a particular reference system with the coordinates of the three particles as \( x_1, x_2 \) and \( x_3 \), the corresponding Jacobi coordinates are:

\[
\begin{align*}
r_1 &= x_2 - x_3, \\
R_1 &= x_1 - \frac{m_2 x_2 + m_3 x_3}{m_2 + m_3}, \\
r_2 &= x_3 - x_1, \\
R_2 &= x_2 - \frac{m_1 x_1 + m_3 x_3}{m_1 + m_3}, \\
r_3 &= x_1 - x_2, \\
R_3 &= x_3 - \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}, \\
R_{CM} &= \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3}.
\end{align*}
\]

In the computation of matrix elements we need to express the Jacobi coordinates of one channel in function of the others:

\[
\begin{align*}
r_i &= \alpha_{ij} r_j + \beta_{ij} R_j \\
R_i &= \gamma_{ij} r_j + \delta_{ij} R_j
\end{align*}
\]
Rearrangement channels and Jacobi coordinates (II)

\[ \alpha_{12} = -\frac{m_1}{m_1 + m_3}, \quad \beta_{12} = +1, \quad \gamma_{12} = -\frac{m_3(m_1 + m_2 + m_3)}{(m_1 + m_3)(m_2 + m_3)}, \quad \delta_{12} = -\frac{m_2}{m_2 + m_3}; \]

\[ \alpha_{13} = -\frac{m_1}{m_1 + m_2}, \quad \beta_{13} = -1, \quad \gamma_{13} = +\frac{m_2(m_1 + m_2 + m_3)}{(m_1 + m_2)(m_2 + m_3)}, \quad \delta_{13} = -\frac{m_3}{m_2 + m_3}; \]

\[ \alpha_{21} = -\frac{m_2}{m_2 + m_3}, \quad \beta_{21} = -1, \quad \gamma_{21} = +\frac{m_3(m_1 + m_2 + m_3)}{(m_1 + m_3)(m_2 + m_3)}, \quad \delta_{21} = -\frac{m_1}{m_1 + m_3}; \]

\[ \alpha_{23} = -\frac{m_2}{m_1 + m_2}, \quad \beta_{23} = +1, \quad \gamma_{23} = -\frac{m_1(m_1 + m_2 + m_3)}{(m_1 + m_2)(m_1 + m_3)}, \quad \delta_{23} = -\frac{m_3}{m_1 + m_3}; \]

\[ \alpha_{31} = -\frac{m_3}{m_2 + m_3}, \quad \beta_{31} = +1, \quad \gamma_{31} = +\frac{m_2(m_1 + m_2 + m_3)}{(m_1 + m_2)(m_2 + m_3)}, \quad \delta_{31} = +\frac{m_1}{m_1 + m_2}; \]

\[ \alpha_{32} = -\frac{m_3}{m_1 + m_3}, \quad \beta_{32} = -1, \quad \gamma_{32} = +\frac{m_1(m_1 + m_2 + m_3)}{(m_1 + m_2)(m_1 + m_3)}, \quad \delta_{32} = -\frac{m_2}{m_1 + m_2}. \]
We consider how to calculate a central-potential matrix element of the following type (Spin and isospin parts are omitted for simplicity of expressions):

\[
\langle \Psi^{(k)}_{JM, \alpha_k n_k N_k} (r_k, R_k) | V(r_j) | \Psi^{(i)}_{JM, \alpha_i n_i N_i} (r_i, R_i) \rangle
\]

in which the ket- and bra-vectors are from different channels \( i \) and \( k \) and the potential is a function of \( r_j \).

We transform both the \( i \)-channel and \( k \)-channel functions into \( j \)-channel functions and perform the integration over \( r_j \) and \( R_j \).

\[
\begin{align*}
    r_i &= \alpha_{ij} r_j + \beta_{ij} R_j, \\
    R_i &= \gamma_{ij} r_j + \delta_{ij} R_j \\
    r_k &= \alpha_{kj} r_j + \beta_{kj} R_j, \\
    R_k &= \gamma_{kj} r_j + \delta_{kj} R_j
\end{align*}
\]

Using the formula

\[
\begin{align*}
    r_i^{l_i} Y_{l_i m_i} (\hat{r}_i) &= \sum_{\lambda=0}^{l_i} \left[ \binom{2l_i}{2\lambda} \frac{4\pi(2l_i + 1)}{(2\lambda + 1)(2(l_i - \lambda) + 1)} \right]^{\frac{1}{2}} (\alpha_{ij} r_j)^{l_i - \lambda} (\beta_{ij} R_j)^{\lambda} \times \\
    &\times \left[ Y_{l_i - \lambda} (\hat{r}_j) \otimes Y_{\lambda} (\hat{R}_j) \right]_{l_i m_i}
\end{align*}
\]
We can rewrite the $i$-channel three body basis function as a function of $r_j$ and $R_j$

\[
\left[ \phi_{n_i, l_i}^G(r_i) Y_{l_i}(\hat{r}_i) \otimes \phi_{N_i, L_i}^G(R_i) Y_{L_i}(\hat{R}_i) \right]_{LM_L} = N_{n_i, l_i} N_{N_i, L_i} e^{-\nu_i r_i^2 - \lambda N_i R_i^2} \times
\]

\[
\times \sum_{l_j, L_j} \sum_{K=0}^{l_i + L_i} \langle l_i L_i L_j L_j L; K \rangle_{i \rightarrow j} r_j^{l_i + L_i - K} R_j^K \left[ Y_{l_j}(\hat{r}_j) Y_{L_j}(\hat{R}_j) \right]_{LM_L}
\]

where

\[
\langle l_i L_i L_j L_j L; K \rangle_{i \rightarrow j} = (2l_i + 1)(2L_i + 1) \sum_{\lambda=0}^{l_i} \sum_{\Lambda=0}^{L_i} \left[ \binom{2l_i}{2\lambda} \binom{2L_i}{2\Lambda} \right]^{1/2} \alpha_{ij}^{l_i - \lambda} \beta_{ij}^{\lambda} \gamma_{ij}^{L_i - \Lambda} \delta_{ij}^{\Lambda} \times
\]

\[
\times \left\{ \begin{array}{ccc}
  l_i - \lambda & L_i - \Lambda & l_j \\
  \lambda & \Lambda & L_i \\
  l_i & L_i & L
\end{array} \right\} (l_i - \lambda 0 L_i - \Lambda 0 | l_j 0)(\lambda 0 \Lambda 0 | L_j 0)
\]

and can be calculated and stored prior to the computation.

We still have to deal with the Gaussian part of the expression above:

\[
e^{-\nu_i r_i^2 - \lambda N_i R_i^2} = e^{-\eta_{ij} r_j^2 - \zeta_{ij} R_j^2} \sum_{l=0}^{\infty} (4\pi)^{3/2} 2^{l} \frac{\sqrt{2l + 1}}{\Gamma(l + 1)} I_l(2\xi_{ij} r_j R_j) \left[ Y_l(\hat{r}_j) Y_l(\hat{R}_j) \right]_0
\]

where $I_l(z) = (-i)^l j_l(iz)$ is the modified spherical Bessel function of the first kind and

\[
\eta_{ij} = \nu_i \alpha_{ij}^2 + \lambda N_i \gamma_{ij}^2, \quad \zeta_{ij} = \nu_i \beta_{ij}^2 + \lambda N_i \delta_{ij}^2, \quad \xi_{ij} = \nu_i \alpha_{ij} \beta_{ij} + \lambda N_i \gamma_{ij} \delta_{ij}
\]
The Four-body total wave function $\Psi_{JM}$ is described as a sum of the components of 18 rearrangement channels.

Calculation of the Hamiltonian matrix elements becomes much laborious!
In order to make the calculation tractable, replace the Gaussian basis function by a superposition of infinitesimally-shifted Gaussians:

\[ \phi^G_{nlm}(r) = N_{nl} \lim_{\epsilon \to 0} \frac{1}{(\nu_n \epsilon)^l} \sum_{k=1}^{k_{\text{max}}} C_{lm,k} e^{-\nu_n(r-\epsilon D_{lm,k})^2} \]

where the limit \( \epsilon \to 0 \) must be carried out after the matrix elements have been calculated analytically.

This new set of basis functions make the calculation of three- and four-body matrix elements very easy:

- Advantages of using the usual Gaussians remain with the new basis functions.
- No complicated angular-momentum algebra is needed.

More explicitly:

\[ N_{nl} r^l e^{-\nu_n r^2} Y_{lm}(\hat{r}) = N_{nl} \lim_{\epsilon \to 0} \left( \frac{l}{4 \nu_n \epsilon} \right)^{\frac{l-m}{2}} \sum_{j=0}^{l-m} A_{lm,j} \sum_{s=0}^{p} \sum_{t=0}^{q} \sum_{u=0}^{j} \binom{p}{s} \binom{q}{t} \binom{j}{u} e^{-\nu_n(r-\epsilon D)^2} \]

where

\[ A_{lm,j} = \left[ \frac{(2l + 1)(l - m)!}{4\pi(l + m)!} \right]^{\frac{1}{2}} \frac{(l + m)!}{2^m} \frac{(-1)!}{4j!(m + j)!(l - m - 2j)!} \]

and with

\[ p = l - m - 2j, \quad q = l + m, \quad D = \frac{2}{l} \left[ (2s - p)a_z + (2t - q)a_{xy} + (2u - j)a_{xy}^* \right] \]

To lowest order the decay is described by the matrix element of the QCD interaction Hamiltonian between a hybrid wave function and a two-mesons wave function:

$$\text{Hybrid} \rightarrow \text{meson} + \text{meson}$$

The Hamiltonian annihilating a gluon and creating a quark pair is:

$$H_{\text{int}} = g \int d\mathbf{x} \bar{\psi}(\mathbf{x}) \gamma_{\mu} \frac{\lambda^a}{2} \psi(\mathbf{x}) A_{\alpha}^\mu(\mathbf{x}).$$

We expand at $t = 0$:

$$\psi(\mathbf{x}) = \sum_{s=1}^{2} \int \frac{d\mathbf{p}}{(2\pi)^3} e^{i\mathbf{p}\mathbf{x}} \left[ u_p s b_p s + v_{-p} d_{-p}^\dagger s \right],$$

$$A_{\alpha}^\mu(\mathbf{x}) = \sum_{\lambda=1}^{2} \int \frac{d\mathbf{k}}{\sqrt{2\omega(2\pi)^3}} \varphi_{a\varepsilon_{k\lambda}^{\mu}} \left[ a_{k\lambda} e^{i\mathbf{k}\mathbf{x}} + a_{k\lambda}^\dagger e^{-i\mathbf{k}\mathbf{x}} \right].$$

Then, the transition operator is given by

$$H = g \sum_{ss'} \lambda_0 \int \frac{d\mathbf{p} d\mathbf{k} d\mathbf{p}'}{\sqrt{2\omega(2\pi)^3}} \delta^{(3)}(\mathbf{p} - \mathbf{k} - \mathbf{p}') \bar{u}_p s b_{p s} \gamma_{\mu} \frac{\lambda^a}{2} u_{-p' s'} a_{k\lambda}^\dagger \varphi_{a\varepsilon_{k\lambda}^{\mu}}$$

Once the meson and hybrid wave functions are expanded in function of quark, antiquark and gluon creator operators, the matrix element is:

$$\langle BC | H | A \rangle = g f(A, B, C) (2\pi)^3 \delta^{(3)}(\mathbf{p}_A - \mathbf{p}_B - \mathbf{p}_C)$$
Hybrid decay (II)


with

\[ f(A, B, C) = \sum_{m_{q\bar{q}}, m_g, m_B, m_C, \mu_{q\bar{q}}, \mu_g, \mu_B, \mu_C} \Omega \, X(\mu_{q\bar{q}} \mu_g; \mu_B, \mu_C) \, I(m_{q\bar{q}}, m_g; m_B, m_C, m) \times \]

\[ \times C_{Lg \, m_{g \mu_l} \mu_g} ^{gM_g} \, C_{Lqg \, m_{q\bar{q}g} \mu_{q\bar{q}}} ^{Ln} \, C_{LMs_{q\bar{q}} \mu_{q\bar{q}}} ^{JM} \, C_{Lm_{s_{q\bar{q}g}} \mu_{q\bar{q}}} ^{JM} \, C_{Lm_{s_{gBS}} \mu_{gBS}} ^{JM} \, C_{Lm_{s_{gCS}} \mu_{gCS}} ^{JM} \, \Phi \]

- **Color factor:**
  \[ \Omega = \frac{1}{24} \sum_a \text{Tr}(\lambda^a)^2 = \frac{2}{3} \]

- **Spin factor:**
  \[ X(\mu_{q\bar{q}} \mu_g; \mu_B, \mu_C) = \sum_s \sqrt{2} \sqrt{(2S_B + 1)(2S_C + 1)3(2S_{q\bar{q}} + 1)} \left\{ \begin{array}{ccc} 1/2 & 1/2 & S_B \\ 1/2 & 1/2 & S_C \\ S_{q\bar{q}} & 1 & S \end{array} \right\} \]

\[ \times C^{S_{q\bar{q}}} \mu_{q\bar{q}} \mu_{q\bar{q}g} \mu_{q\bar{q}1} \mu_g \, C^{S_{gBS}} \mu_{gBS} \mu_{gCS} \mu_{gCS} \]

- **Isospin factor:**
  \[ \Phi = \sqrt{(2I_A + 1)(2I_B + 1)(2I_C + 1)} \left\{ \begin{array}{ccc} i_1 & i_3 & I_B \\ i_2 & i_4 & I_C \\ I_A & 0 & I_A \end{array} \right\} \eta \varepsilon \]

- The I’s (i’s) labels the hadron (quark) isospins.
- \( \eta = 1 \) if the gluon goes into strange quarks and \( \eta = \sqrt{2} \) if it goes into non-strange ones.
- \( \varepsilon \) is the number of diagrams contributing to the decay.
Hybrid decay (III)

Spatial overlap factor:

\[ I(m_{q\bar{q}}, m_g, m_B, m_C, m) = \int \int \frac{d\mathbf{p} d\mathbf{k}}{\sqrt{2\omega(2\pi)^6}} \psi_{q\bar{q}}^{m_{q\bar{q}}} (\mathbf{p}_B - \mathbf{p}) \psi_{g}^{m_g} (\mathbf{k}) \]

\[ \times \psi_{B}^{m_B^*} \left( \frac{\mathbf{p}_B}{2} - \mathbf{p} - \frac{\mathbf{k}}{2} \right) \psi_{C}^{m_C^*} \left( -\frac{\mathbf{p}_B}{2} + \mathbf{p} - \frac{\mathbf{k}}{2} \right) d\Omega_B Y_{l}^{m^*}(\Omega_B) \]

Selection rules:

- Factor \( \delta_{l g 0} \). TE hybrid cannot decay into two mesons which have both zero orbital angular momentum between their quarks.

- Factor \( \delta_{l g 0} \). TM hybrids with \( j_g \geq 2 \) cannot decay into ground mesons pseudoscalar-pseudoscalar, pseudoscalar-vector and vector-vector.

- Factor \( \delta_{l q\bar{q} l} \). The inter-meson orbital momentum is a direct measure of the interquark orbital momentum in the hybrid.

Keep in mind that the selection rules are model dependent and this applies to other theoretical statements that appear in the literature and are considered dogma

Other models like the flux-tube decay model (\(^3P_0\) model + Gaussian vertex) or the PSS hybrid decay model (\(^3S_1\) model) can be explored. Common predictions:

- Low-lying hybrids do not decay to two identical mesons.

- The pair-creation is in a spin-triplet. It appears to be a universal feature in all nonrelativistic decay models.
A new generation of facilities are scheduled for the next two decades: run II LHCb@CERN, GlueX@JLab12 and PANDA@FAIR. One of their main goals is to produce in the primary collision point a rich environment of gluons in order to promote the formation of glueballs and quark-gluon hybrids.

If a low energy gluonic spectroscopy can be discovered and decoded, this will change the way we think that matter is constructed. For the first time, it will be shown that the QCD’s gauge boson (gluon) participates at the same level than the basic fermions of the theory (quarks) in building matter.

A new breed of models that is capable of reproducing central lattice results is required

- These models will reproduce the gluonic adiabatic potentials and the spectrum of heavy and light hybrids reasonably well.
- This will require a formalism that captures short range and long range dynamics in an approximate fashion without double counting or other conceptual issues.
- Such a model should also be able to describe strong and electromagnetic decays reasonably accurately.

Experimental outcomes and theoretical insights should hopefully lead to a quantitative and qualitative understanding of the soft gluonic sector of the Standard Model.