On the $J/\psi \to \gamma \eta_c \to \gamma X$ line-shape in pNRQCD

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Abstract

We study the photon spectrum line-shape of the $J/\psi \to \gamma \eta_c \to \gamma X$ decay process in a nonrelativistic effective field theory (EFT) framework of quarkonium photodisintegration (pNRQCD). We take into account the finite width of the $\eta_c$ and include $O(\alpha_s^2)$ corrections. We observe that the photon spectrum line-shape is divergent at large energies due to polynomially and logarithmically divergent terms, that upon integration over the photon energies in dimensional regularization (DR) produce no contribution or can be renormalized. We propose to subtract these divergences at the line-shape level in a manner consistent with the calculation of the width in DR and $\overline{\text{MS}}$ scheme. We analyze CLEO’s data with the proposed subtracted line-shape and find good agreement between the theoretical prediction and the experimental result.

Experimental situation

The $J/\psi \to \eta_c$ branching fraction was first measured in 1986 by the Crystal Ball Collaboration in the inclusive photon spectrum and the value $B_{J/\psi \to \eta_c} = 0.05 \pm 0.04 \pm 0.01$ was obtained. The transition was not measured again until 2009 by the CLEO Collaboration [1], which analyzed all reported $\eta_c$ decay modes except the $p\bar{p}$ one due to its small rate. The CLEO Collaboration measured $B_{J/\psi \to \eta_c} = 0.042 \pm 0.008 \pm 0.010$ [3]. More recently the KEDR Collaboration [2, 3] measured the transition using the inclusive photon spectrum and reported $B_{J/\psi \to \eta_c} = 0.040 \pm 0.015 \pm 0.017$ [3]. Note that this value is not taken into account in the Particle Data Group (PDG) average [4], which considers only the Crystal Ball and CLEO measurements.

One of the crucial ingredients in the determination of the branching fraction from experimental measurements is the photon spectrum line-shape used in the analysis. The line-shape is fitted together with the background to the experimental data and the number of events above background is used to determine the branching fraction. The CLEO Collaboration Ref. [1] observed for the first time a clear asymmetry in the photon energy spectrum line-shape due to phase-space and energy-dependent terms in the $J/\psi \to \eta_c$ transition matrix element [5]. In order to obtain a good fit to the data, the photon spectrum line-shape was constructed with a relativistic Breit-Wigner distribution modified by a factor $k^3$, where $k$ is the photon energy. However, varying this factor $k$ and a divergent tail at large photon energies. In order to suppress this behaviour, an ad hoc damping function was included, arguing that it modeled the overlap of the charmonium wave functions. Nevertheless, such damping factor does not appear in the theoretical studies of Refs. [5, 6] and thus it is not well justified. The analysis by the KEDR Collaboration [2] followed a similar approach incorporating a different, non-theoretically motivated, damping function.

Subtracted line-shape in pNRQCD

The $J/\psi \to \eta_c$ transition amplitude has been computed in Ref. [5] within the weakly-coupled pNRQCD approach and can be written, up to $O(\alpha_s^3)$, as

$$ A_{J/\psi \to \eta_c} = \frac{\mathcal{B}_{J/\psi \to \eta_c}}{\mathcal{B}_{J/\psi \to \gamma \eta_c}}(k^2 + \xi^2)^{\alpha_0} \left( 1 + \frac{5}{2} \xi^2 \right) \left( 1 + \xi^2 \right)^3, $$

where $k$ is the photon momentum, $\xi$, a constant of the model, is the photon momentum squared divided by the mass squared of the $\eta_c$, and $\mathcal{B}_{J/\psi \to \eta_c}$ is the branching fraction of $J/\psi \to \eta_c$.

The photon spectrum line-shape read as follows

$$ d^2\sigma_{\text{UV div.}} = \frac{\mathcal{B}_{J/\psi \to \eta_c}}{\mathcal{B}_{J/\psi \to \gamma \eta_c}} \frac{\mathcal{B}_{J/\psi \to \gamma \eta_c}}{\mathcal{B}_{J/\psi \to \gamma X}} \frac{1}{2 \pi} \frac{d\phi}{k^2} \left( \frac{k^2 + \xi^2}{k^2} \right)^{\alpha_0} \left( 1 + \frac{5}{2} \xi^2 \right) \left( 1 + \xi^2 \right)^3 d^2k. $$

To regularize spurious infrared divergences produced by the separation of the logarithmic divergence we have introduced a regulator $\Delta$ in the last term of Eq. (3). Setting this regulator to be the logarithmic divergence is subtracted at the integrand level has been developed in Ref. [6], where the subtraction of the photon spectrum line-shape read as follows

$$ d^2\sigma_{\text{pNRQCD}} = d^2\sigma_{\text{UV div.}} - d^2\sigma_{\text{IR}}. $$

Since any polynomially divergent term vanishes upon integration in DR, one could in principle choose a subtraction different from Eq. (3) by any amount of polynomially divergent terms and it would still produce the same renormalized decay width. Nevertheless, only the subtraction in Eq. (5) leaves the photon spectrum line-shape free of any IR and UV divergences.

Analysis of CLEO’s data

The photon spectrum for the $J/\psi \to \eta_c \to \gamma X$ process measured by the CLEO Collaboration is shown in Fig. 2. We have fitted the unsubtracted (panel (a)) and subtracted (panel (b)) and Eq. (4) pNRQCD line-shapes with a free normalization together with the CLEO’s background shape. The total signal is convoluted with a resuspension factor for width of 85.3 MB. In both cases, we set $d = 4$, take $\eta_c = 300 \pm 200$ MeV and $\rho = 0.33 \pm 0.05$ to be the subtracted line-shape. The remaining parameters, such as charm quark mass, have been specified before. It can be seen in Fig. 2 that a substantial tail of signal persists at large photon energies when fitting CLEO’s data with the subtracted pNRQCD line-shape, while using the subtracted line-shape removes that tail. This produces a significantly different value for $\mathcal{B}_{J/\psi \to \gamma \eta_c}$ branch fraction.

In Table 1, we present our results obtained from the analysis of CLEO’s data for the $\mathcal{B}_{J/\psi \to \gamma \eta_c}$ and the branching fraction $\mathcal{B}_{J/\psi \to \gamma X}$.

Conclusions

We have calculated the photon spectrum line-shape for the $J/\psi \to \eta_c \to \gamma X$ process in weakly-coupled pNRQCD [7]. We have incorporated to the leading order expression for the $J/\psi \to \eta_c$ transition amplitude the relativistic and multipole expansion corrections up to $O(\alpha_s^3)$ [7]. These corrections have been computed incorporating the full static potential into the leading order Hamiltonian as in Ref. [6].

We have argued that the large energy tail of the line-shape is due to either polynomially or logarithmically divergent terms. We have analyzed CLEO’s data for $J/\psi \to \eta_c \to \gamma X$ process using a photon spectrum line-shape in which the UV divergent terms are subtracted in a manner consistent with the calculation of the decay width in DR and $\overline{\text{MS}}$ scheme. Using the subtracted line-shape in the analysis of the experimental data, the experimental and theoretical determinations of the branching fraction are in good agreement.

References