Nucleon, Delta and Nucleon to Delta Electromagnetic Form Factors in DSEs

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Quantum Chromodynamics is the only known example in nature of a nonperturbative fundamental quantum field theory.

QCD have profound implications for our understanding of the real-world:
- Explain how quarks and gluons bind together to form hadrons.
- Origin of the 98% of the mass in the visible universe.

Given QCD’s complexity:
- The best promise for progress is a strong interplay between experiment and theory.

Emergent phenomena

Quark and gluon confinement

Dynamical chiral symmetry breaking

Colored particles have never been seen isolated

Hadrons do not follow the chiral symmetry pattern

Neither of these phenomena is apparent in QCD’s Lagrangian yet!
They play a dominant role determining characteristics of real-world QCD.
Emergent phenomena: Confinement

**Confinement is associated with dramatic, dynamically-driven changes in the analytic structure of QCD’s propagators and vertices (QCD’s Schwinger functions)**

☞ **Dressed-propagator for a colored state:**

- An observable particle is associated with a pole at timelike-$P^2$.
- When the dressing interaction is confining: Real-axis mass-pole splits, moving into a pair of complex conjugate singularities.
- No mass-shell can be associated with a particle whose propagator exhibits such singularity.

☞ **Dressed-gluon propagator:**

- Confined gluon.
- IR-massive but UV-massless.
- $m_G \sim 2 - 4\Lambda_{QCD}$ ($\Lambda_{QCD} \simeq 200 \text{ MeV}$).

Any 2-point Schwinger function with an inflexion point at $p^2 > 0$:

→ **Breaks the axiom of reflexion positivity**

→ **No physical observable related with**
**Spectrum of a theory invariant under chiral transformations should exhibit degenerate parity doublets**

\[
\begin{align*}
\pi & \quad J^P = 0^- \quad m = 140 \text{ MeV} \quad \text{cf.} \quad \sigma & \quad J^P = 0^+ \quad m = 500 \text{ MeV} \\
\rho & \quad J^P = 1^- \quad m = 775 \text{ MeV} \quad \text{cf.} \quad a_1 & \quad J^P = 1^+ \quad m = 1260 \text{ MeV} \\
N & \quad J^P = 1/2^+ \quad m = 938 \text{ MeV} \quad \text{cf.} \quad N(1535) & \quad J^P = 1/2^- \quad m = 1535 \text{ MeV}
\end{align*}
\]

**Splittings between parity partners are greater than 100-times the light quark mass scale:** \( m_u/m_d \sim 0.5, \ m_d = 4 \text{ MeV} \)

- **Dynamical chiral symmetry breaking**
  - Mass generated from the interaction of quarks with the gluon-medium.
  - Quarks acquire a **HUGE** constituent mass.
  - Responsible of the 98% of the mass of the proton.

- **(Not) spontaneous chiral symmetry breaking**
  - Higgs mechanism.
  - Quarks acquire a **TINY** current mass.
  - Responsible of the 2% of the mass of the proton.
Confinement and Dynamical Chiral Symmetry Breaking (DCSB) can be identified with properties of dressed-quark and -gluon propagators and vertices (Schwinger functions).

**Dyson-Schwinger equations (DSEs)**

- The quantum equations of motion of QCD whose solutions are the Schwinger functions.
  - Propagators and vertices.

- Generating tool for perturbation theory.
  - No model-dependence.

- Nonperturbative tool for the study of continuum strong QCD.
  - Any model-dependence should be incorporated here.

- Allows the study of the interaction between light quarks in the whole range of momenta.
  - Analysis of the infrared behaviour is crucial to disentangle confinement and DCSB.

- Connect quark-quark interaction with experimental observables.
  - Elastic and transition form factors can be used to illuminate QCD (at infrared momenta).
The simplest example of DSEs: The gap equation

\[ S^{-1}(p) = Z_2(i\gamma \cdot p + m^{\text{bm}}) + \Sigma(p) \]

\[ \Sigma(p) = Z_1 \int_q g^2 D_{\mu\nu}(p-q) \frac{\lambda^a}{2} \gamma\mu S(q) \frac{\lambda^a}{2} \Gamma_\nu(q, p) \]

General solution:

\[ S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)} \]

Kernel involves:
- \( D_{\mu\nu}(p-q) \) - dressed gluon propagator
- \( \Gamma_\nu(q, p) \) - dressed-quark-gluon vertex

Each of which satisfies its own Dyson-Schwinger equation

\[ \text{Infinitely many coupled equations} \]

Coupling between equations necessitates truncation

Rapid acquisition of mass is effect of gluon cloud

\[ M(p^2) \text{ exhibits dynamical mass generation} \]
Vector and Axial-vector Ward-Green-Takahashi identities

Symmetries should be preserved by any truncation

Highly nontrivial constraint → failure implies loss of any connection with QCD

Symmetries in QFT are implemented by WTIs which relate different Schwinger functions

- For instance, axial-vector Ward-Takahashi identity:

These observations show that symmetries relate the kernel of the gap equation – a one-body problem – with that of the Bethe-Salpeter equation – a two-body problem –
Mesons

A 2-body bound state problem in quantum field theory.

Properties emerge from solutions of Bethe-Salpeter equation:

\[
\Gamma(k; P) = \int \frac{d^4q}{(2\pi)^4} K(q, k; P) S(q+P) \Gamma(q; P) S(q)
\]

The kernel is that of the gap equation.

Baryons

A 3-body bound state problem in quantum field theory.

Structure comes from solving the Faddeev equation.

**Faddeev equation:** Sums all possible quantum field theoretical interactions that can take place between the three quarks that define its valence quark content.
Diquarks inside baryons

The attractive nature of quark-antiquark correlations in a color-singlet meson is also attractive for $\bar{3}_c$ quark-quark correlations within a color-singlet baryon.

Diquark correlations:

- Empirical evidence in support of strong diquark correlations inside the nucleon.
- A dynamical prediction of Faddeev equation studies.
- In our approach: Non-pointlike color-antitriplet and fully interacting.

Thanks to G. Eichmann.

Diquark composition of the nucleon and $\Delta$

- **Positive parity states**
  - Pseudoscalar and vector diquarks
    - Ignored
      - Wrong parity
      - Larger mass-scales
  - Scalar and axial-vector diquarks
    - Dominant
      - Right parity
      - Shorter mass-scales

$N \Rightarrow 0^+, 1^+$ diquarks
$\Delta \Rightarrow$ only $1^+$ diquark
A central goal of Nuclear Physics: understand the structure and properties of protons and neutrons, and ultimately atomic nuclei, in terms of the quarks and gluons of QCD.

Elastic and transition form factors

- Unique window into its quark and gluon structure
- High-$Q^2$ reach by experiments
- Distinctive information on the roles played by confinement and DCSB in QCD
- Probe the excited nucleon structures at perturbative and non-perturbative QCD scales

CEBAF Large Acceptance Spectrometer (CLAS)

- Most accurate results for the electroexcitation amplitudes of the four lowest excited states.
- They have been measured in a range of $Q^2$ up to:
  - 8.0 GeV$^2$ for $\Delta(1232)P_{33}$ and $N(1535)S_{11}$.
  - 4.5 GeV$^2$ for $N(1440)P_{11}$ and $N(1520)D_{13}$.
- The majority of new data was obtained at JLab.

Upgrade of CLAS up to 12 GeV → CLAS12 (New generation experiments in 2015)
The $\gamma^* N \rightarrow \Delta$ reaction

Two ways in order to analyze the structure of the $\Delta$-resonances

- $\pi$-mesons as a probe
  - complex
- Photons as a probe
  - relatively simple

**BUT:** $\mathcal{B}(\Delta \rightarrow \gamma N) \lesssim 1\%$

*This became possible with the advent of intense, energetic electron-beam facilities*

- Reliable data on the $\gamma^* p \rightarrow \Delta^+$ transition:
  - Available on the entire domain $0 \leq Q^2 \lesssim 8 \text{GeV}^2$.
  - Isospin symmetry implies $\gamma^* n \rightarrow \Delta^0$ is simply related with $\gamma^* p \rightarrow \Delta^+$.

$\gamma^* p \rightarrow \Delta^+$ data has stimulated a great deal of theoretical analysis:

- Deformation of hadrons.
- The relevance of pQCD to processes involving moderate momentum transfers.
- The role that experiments on resonance electroproduction can play in exposing non-perturbative phenomena in QCD:
  - The nature of confinement and Dynamical Chiral Symmetry Breaking.
The electromagnetic current can be generally written as:

\[ J_{\mu\lambda}(K, Q) = \Lambda^\pm(P_f) R_{\lambda\alpha}(P_f) i\gamma_5 \Gamma_{\alpha\mu}(K, Q) \Lambda^\pm(P_i) \]

- Incoming nucleon: \( P_i^2 = -m_N^2 \), and outgoing delta: \( P_f^2 = -m_\Delta^2 \).
- Photon momentum: \( Q = P_f - P_i \), and total momentum: \( K = (P_i + P_f)/2 \).
- The on-shell structure is ensured by the \( N \)- and \( \Delta \)-baryon projection operators.

The composition of the 4-point function \( \Gamma_{\alpha\mu} \) is determined by Poincaré covariance:

Convenient to work with orthogonal momenta \( \leftrightarrow \) Simplify its structure considerably

Not yet the case for \( K \) and \( Q \) \( \leftrightarrow \) \( \Delta(m_\Delta - m_N) \neq 0 \Rightarrow K \cdot Q \neq 0 \)

We take instead \( \hat{K}_\mu^\perp = T^Q_{\mu\nu} \hat{K}_\nu \) and \( \hat{Q} \)

Vertex decomposes in terms of three (Jones-Scadron) form factors:

\[ \Gamma_{\alpha\mu} = \hat{\kappa} \left[ \frac{\lambda_m}{2\lambda_+} (G_M^* - G_E^*) \gamma_5 \epsilon_{\alpha\mu\gamma\delta} \hat{K}_\gamma^\perp \hat{Q}_\delta - G_E^* T^Q_{\alpha\gamma} T^K_{\gamma\mu} - \frac{i\varsigma}{\lambda_m} G_C^* \hat{Q}_\alpha \hat{K}_\mu^\perp \right] , \]

Magnetic dipole \( \Rightarrow G_M^* \)
Electric quadrupole \( \Rightarrow G_E^* \)
Coulomb quadrupole \( \Rightarrow G_C^* \)
The Jones-Scadron form factors are:

\[ G_M^* = 3(s_2 + s_1), \]
\[ G_E^* = s_2 - s_1, \]
\[ G_C^* = s_3. \]

The scalars are obtained from the following Dirac traces and momentum contractions:

\[ s_1 = n \frac{\sqrt{\varsigma(1 + 2d)}}{d - \varsigma} \Gamma^K_{\mu \nu} \hat{K}_\lambda^\perp \text{Tr}[\gamma_5 J_{\mu \lambda} \gamma_{\nu}], \]
\[ s_2 = n \frac{\lambda^+}{\lambda_m} \Gamma^K_{\mu \lambda} \text{Tr}[\gamma_5 J_{\mu \lambda}], \]
\[ s_3 = 3n \frac{\lambda^+ (1 + 2d)}{d - \varsigma} \hat{K}_\mu \hat{K}_\lambda^\perp \text{Tr}[\gamma_5 J_{\mu \lambda}]. \]

We have used the following notation:

\[ n = \frac{\sqrt{1 - 4d^2}}{4i k \lambda_m}, \quad \lambda^\pm = \frac{(m_\Delta \pm m_N)^2 + Q^2}{2(m_\Delta^2 + m_N^2)}, \quad \varsigma = \frac{Q^2}{2(m_\Delta^2 + m_N^2)}, \]
\[ d = \frac{m_\Delta^2 - m_N^2}{2(m_\Delta^2 + m_N^2)}, \quad \lambda_m = \sqrt{\lambda^+ \lambda^-}, \quad k = \sqrt{\frac{3}{2}} \left(1 + \frac{m_\Delta}{m_N}\right). \]

Experimental results and theoretical expectations


The $R_{EM}$ ratio is measured to be minus a few percent.

The $R_{SM}$ ratio does not seem to settle to a constant at large $Q^2$.

**pQCD predictions**

For $Q^2 \to \infty$
- $G_M^* \to 1/Q^4$.
- $R_{EM} \to +100\%$.
- $R_{SM} \to$ constant.

**CQM predictions**

Without quark orbital angular momentum:
- $R_{EM} \to 0$.
- $R_{SM} \to 0$.

**SU(6) predictions**

\[
\langle p|\mu|\Delta^+ \rangle = \langle n|\mu|\Delta^0 \rangle = -\sqrt{2} \langle n|\mu|n \rangle
\]

Data do not support these predictions

Our aim: try to understand this longstanding puzzle
Electromagnetic current description in the quark-diquark picture

To compute the electromagnetic properties of the $\gamma^* N\Delta$ reaction in a given framework, one must specify how the photon couples to its constituents.

**Six contributions to the current**

- Coupling of the photon to the dressed quark.
- Coupling of the photon to the dressed diquark:
  - Elastic transition.
  - Induced transition.
- Exchange and seagull terms.

**Ingredients in the contributions**

- $\Psi_{i,f}$ ≡ Faddeev amplitudes.
- Single line ≡ Quark prop.
- Double line ≡ Diquark prop.
- $\Gamma$ ≡ Diquark BS amplitudes.

**One-loop diagrams**

- $P_i$ $\Psi_i$ $\Psi_f$ $P_i$
- $Q$ $\Gamma$

**Two-loop diagrams**

- $P_i$ $\Psi_f$ $\Psi_i$ $P_i$
- $Q$ $\Gamma$
- $X_{\mu}$ $\Gamma$ $\Psi_i$ $P_i$

- $P_i$ $\Psi_f$ $\Psi_i$ $P_i$
- $Q$ $\Gamma$
- $X_{\mu}$ $\Gamma$ $\Psi_i$ $P_i$
Each diagram can be expressed in a similar way:

\[ \Gamma_\mu = \Lambda_+(P_f) J_\mu(K, Q) \Lambda_+(P_i) \]

Photon coupling directly to a dressed-quark with the diquark acting as a bystander

Initial state and final state: Proton

- Two axial-vector diquark isospin states:
  - \((I, I_z) = (1, 1) \rightarrow \text{flavor content: } \{uu\}\)
  - \((I, I_z) = (1, 0) \rightarrow \text{flavor content: } \{ud\}\)

- In the isospin limit, they appear with relative weighting: \((-\sqrt{2/3}) : (\sqrt{1/3})\)

Therefore

\[ J_\mu^{\text{scalar}} = \sqrt{\frac{1}{3}} \sqrt{\frac{1}{3}} e_u I_\mu^{\{ud\}} \neq 0 \]
\[ J_\mu^{\text{axial}} = \sqrt{\frac{2}{3}} \sqrt{\frac{2}{3}} e_d I_\mu^{\{uu\}} + \sqrt{\frac{1}{3}} \sqrt{\frac{1}{3}} e_u I_\mu^{\{ud\}} = 0 \]

Hard contributions appear in the microscopic description of the elastic form factor of the proton
Elastic form factor of the proton in the quark-diquark picture (II)

Remaining diagrams: Photon interacting with diquarks

I.C. Cloet et al., Few Body Syst. 46, 1-36 (2012)

\[ \Psi_i \rightarrow \Psi_f \]

\[ Q \]

\[ P_f \rightarrow \psi_f \rightarrow \psi_i \rightarrow P_i \]

\[ G_{E,M}^Q, G_M^Q, G_E^Q, G_M^Q \]

\[ Q^2 \text{(GeV}^2) \]

\[ r_{qq} \gtrsim r_\pi \]

Soft contributions appear in the microscopic description of the elastic form factor of the proton
Diagrams in which the photon interact with diquarks appear

**Photon coupling directly to a dressed-quark with the diquark acting as a bystander**

- **Initial state:** Proton
  - Two axial-vector diquark isospin states:
    - \((l, l_z) = (1, 1) \rightarrow \text{flavor content: } \{uu\}\)
    - \((l, l_z) = (1, 0) \rightarrow \text{flavor content: } \{ud\}\)
  - In the isospin limit, they appear with relative weighting: \((-\sqrt{2/3}) : (\sqrt{1/3})\)
- **Final state:** \(\Delta^+\)
  - Same isospin states of axial-vector diquark.
  - Different weighting due to \(I_\Delta = 3/2\):
    - \((\sqrt{1/3}) : (\sqrt{2/3})\)
- Therefore
  \[
  J^{1,\text{axial}}_{\mu\alpha} = -\sqrt{\frac{2}{3}}\sqrt{\frac{1}{3}} e_d I_{\mu\alpha}^{1\{uu\}} + \sqrt{\frac{1}{3}}\sqrt{\frac{2}{3}} e_u I_{\mu\alpha}^{1\{ud\}} = \frac{\sqrt{2}}{3} I_{\mu\alpha}^{1\{qq\}} (K, Q)
  \]
General observation

$G_M^p$ vs $G_M^{*p}$

- Similar contributions in both cases:

  \[ G_M^{*p} \text{ should fall asymptotically at the same rate as } G_M^p. \]

- By isospin considerations:

  \[ G_M^{*n} \text{ should fall asymptotically at the same rate as } G_M^{*p}. \]

- Hold SU(6):

  \[ \langle p|\mu|\Delta^+\rangle \propto \langle n|\mu|\Delta^0\rangle \propto \langle p|\mu|p\rangle. \]
Simple framework

**Symmetry preserving Dyson-Schwinger equation treatment of a vector × vector contact interaction**

- **Gluon propagator:** Contact interaction.
  \[
  g^2 D_{\mu\nu}(p - q) = \delta_{\mu\nu} \frac{4\pi\alpha_{\text{IR}}}{m_G^2}
  \]

- **Truncation scheme:** Rainbow-ladder.
  \[
  \Gamma_\nu^a(q, p) = (\lambda^a / 2)\gamma_\nu
  \]

- **Quark propagator:** Gap equation.
  \[
  S^{-1}(p) = i\gamma \cdot p + m + \Sigma(p)
  = i\gamma \cdot p + M
  \]
  - \(M \sim 0.4\text{ GeV}.\) Implies momentum independent constituent quark mass.
  - Implies momentum independent BSAs.

- **Baryons:** Faddeev equation.
  \[
  m_N = 1.14 \text{ GeV} \quad m_\Delta = 1.39 \text{ GeV}
  \]
  (masses reduced by meson-cloud effects)

- **Form Factors:** Two-loop diagrams not incorporated.
  - **Exchange diagram**
    - It is zero because our treatment of the contact interaction model
  - **Seagull diagrams**
    - They are zero
Features and flaws of a contact interaction treatment of the kaon

Spectrum of hadrons with strangeness
C. Chen, L. Chang, C.D. Roberts, S. Wan and D.J. Wilson

Nucleon and Roper electromagnetic elastic and transition form factors
D.J. Wilson, I.C. Cloët, L. Chang and C.D. Roberts

π- and ρ-mesons, and their diquark partners, from a contact interaction
H.L.L. Roberts, A. Bashir, L.X. Gutierrez-Guerrero, C.D. Roberts and D.J. Wilson

Masses of ground and excited-state hadrons
H.L.L. Roberts, L. Chang, I.C. Cloët and C.D. Roberts

Abelian anomaly and neutral pion production

Pion form factor from a contact interaction
L.X. Gutierrez-Guerrero, A. Bashir, I.C. Cloët and C.D. Roberts
A truncation which produces Faddeev amplitudes that are independent of relative momentum:

- Underestimates the quark orbital angular momentum content of the bound-state.
- Eliminates two-loop diagram contributions.

<table>
<thead>
<tr>
<th>$G^*_M(Q^2 = 0)$</th>
<th>ind.-p DSE kernels</th>
<th>dep.-p DSE kernels</th>
</tr>
</thead>
<tbody>
<tr>
<td>axial-diquark($\Delta$) − axial-diquark($p$)</td>
<td>0.85</td>
<td>0.83</td>
</tr>
<tr>
<td>axial-diquark($\Delta$) − scalar-diquark($p$)</td>
<td>0.18</td>
<td>1.10</td>
</tr>
</tbody>
</table>

**Improved version**

- Rescale the axial-diquark($\Delta$) − scalar-diquark($p$) diagram using:
  \[
  1 + \frac{g_{as/aa}}{1 + Q^2/m_p^2}
  \]

  \[
  \text{axial($\Delta$)-scalar($p$)} = \text{axial($\Delta$)-axial($p$)} \quad \text{for} \quad G^*_M(Q^2 = 0)
  \]

- Incorporate dressed quark-anomalous magnetic moment $\Leftrightarrow$ Consequence of the DCSB.

Jorge Segovia (Argonne National Laboratory)
More sophisticated framework

☞ Gluon propagator: $1/k^2$-behaviour.

☞ Form Factors: Two-loop diagrams incorporated.

☞ Truncation scheme: Rainbow-ladder.

$\Gamma_{\nu}^a(q, p) = (\lambda^a/2)\gamma^\nu$

☞ Quark propagator: Gap equation.

$$S^{-1}(p) = Z_2(i\gamma \cdot p + m^{bm}) + \Sigma(p)$$

$$= [1/Z(p^2)] [i\gamma \cdot p + M(p^2)]$$

$M \sim 0.33\text{ GeV}$. Implies momentum dependent constituent quark mass.

☞ Baryons: Faddeev equation.

$m_N = 1.18\text{ GeV} \quad m_\Delta = 1.33\text{ GeV}$

(masses reduced by meson-cloud effects)
Electromagnetic form factors of the Nucleon

Bibliography:
- Current quark mass dependence of nucleon magnetic moments and radii
- Survey of nucleon electromagnetic form factors
- Revealing dressed-quarks via the proton’s charge distribution

Observations:
- Model parameters revisited in order to accomodate $\Delta$ and $N \rightarrow \Delta$.
  - Axial-vector diquark properties.
  - Axial-scalar transition.
- Anomalous magnetic moment of the dressed quark.

Results:
$Q^2$-behaviour of $G^*_M$, Jones–Scadron (I)

$G^*_M,J-S$ cf. Experimental data and dynamical models

- **Solid-black:** Sophisticated interaction.
- **Dashed-blue:** Contact interaction.
- **Dot-Dashed-green:** Dynamical + no meson-cloud

Observations:
- All curves are in marked disagreement at infrared momenta.
- Similarity between **Solid-black** and **Dot-Dashed-green**.
- The discrepancy at infrared comes from omission of meson-cloud effects.
- Both curves are consistent with data for $Q^2 \gtrsim 0.75m^2_\Delta \sim 1.14$ GeV$^2$. 

Jorge Segovia (Argonne National Laboratory)  
Nucleon, Delta and Nucleon to Delta EM Form Factors in DSEs
$$Q^2\text{-behaviour of } G^*_M,\text{Jones–Scadron (II)}$$

**Transition cf. elastic magnetic form factors**

![Graph showing the behavior of $G^*_M$ with $Q^2$.]

- **Solid-black:** Sophisticated interaction.
- **Dashed-blue:** Contact interaction.

**Fall-off rate of** $G^*_M, J_S (Q^2)$ **in the** $\gamma^* p \rightarrow \Delta^+$ **must much that of** $G_M (Q^2)$.

**With isospin symmetry:**

$$\langle p|\mu|\Delta^+\rangle = - \langle n|\mu|\Delta^0\rangle$$

so same is true of the $\gamma^* n \rightarrow \Delta^0$ magnetic form factor.

**These are statements about the dressed quark core contributions**

→ **Outside the domain of meson-cloud effects,** $Q^2 \gtrsim 1 \text{ GeV}^2$
Presentations of experimental data typically use the Ash convention – $G_{M,Ash}^{*}(Q^2)$ falls faster than a dipole –

- No sound reason to expect: $G_{M,Ash}^{*}/G_M \sim \text{constant}$

- Jones-Scadron should exhibit: $G_{M,J-S}^{*}/G_M \sim \text{constant}$

- Meson-cloud effects
  - More than 30% for $Q^2 \lesssim 0.75m_\Delta$.
  - Very soft $\rightarrow$ disappear rapidly.

- $G_{M,Ash}^{*}$ vs $G_{M,J-S}^{*}$
  - A factor $1/Q$ of difference.
**Electric and coulomb quadrupoles**

- $R_{EM} = R_{SM} = 0$ in SU(6)-symmetric CQM.
  - Deformation of the hadrons involved.
  - Modification of the structure of the transition current.

- $R_{SM}$: Good description of the rapid fall at large momentum transfer.

- $R_{EM}$: A particularly sensitive measure of orbital angular momentum correlations.

**Graphs:**

- **Left Graph:**
  - $R_{SM}(%)$ vs. $x = Q^2/m_\Delta^2$

- **Right Graph:**
  - $R_{EM}(%)$ vs. $x = Q^2/m_\Delta^2$

**Zero Crossing in the transition electric form factor**

- **Contact interaction** → at $Q^2 \sim 0.75m_\Delta^2 \sim 1.14$ GeV$^2$
- **Sophisticated interaction** → at $Q^2 \sim 1.25m_\Delta^2 \sim 1.90$ GeV$^2$
Helicity conservation arguments in pQCD should apply equally to an internally-consistent symmetry-preserving treatment of a contact interaction

\[ R_{EM} \xrightarrow{Q^2 \to \infty} 1, \quad R_{SM} \xrightarrow{Q^2 \to \infty} \text{constant} \]

Observations:
- Truly asymptotic \( Q^2 \) is required before predictions are realized.
- \( R_{EM} = 0 \) at an empirical accessible momentum and then \( R_{EM} \to 1 \).
- \( R_{SM} \to \text{constant} \). Curve contains the logarithmic corrections expected in QCD.
The $\Delta$ elastic form factors

The small-$Q^2$ behaviour of the $\Delta$ elastic form factors is a necessary element in computing the $\gamma^* N \to \Delta$ transition form factors

- The electromagnetic current can be generally written as:

$$J_{\mu,\lambda\omega}(K, Q) = \Lambda_+(P_f) R_{\lambda\alpha}(P_f) \Gamma_{\mu,\alpha\beta}(K, Q) \Lambda_+(P_i) R_{\beta\omega}(P_i)$$

- Vertex decomposes in terms of four form factors:

$$\Gamma_{\mu,\alpha\beta}(K, Q) = \left[(F_1^* + F_2^*)i\gamma_\mu - \frac{F_2^*}{m_\Delta} K_\mu\right] \delta_{\alpha\beta} - \left[(F_3^* + F_4^*)i\gamma_\mu - \frac{F_4^*}{m_\Delta} K_\mu\right] \frac{Q_\alpha Q_\beta}{4m_\Delta^2}$$

- The multipole form factors:

$$G_{E0}(Q^2), \quad G_{M1}(Q^2), \quad G_{E2}(Q^2), \quad G_{M3}(Q^2),$$

are functions of $F_1^*, F_2^*, F_3^*$ and $F_4^*$.

- They are obtained by any four sensible projection operators. Physical interpretation:
  
  - $G_{E0}$ and $G_{M1} \to$ Momentum space distribution of $\Delta$’s charge and magnetization.
  - $G_{E2}$ and $G_{M3} \to$ Shape deformation of the $\Delta$-baryon.
Since one must deal with the very short $\Delta$-lifetime ($\tau_\Delta \sim 10^{-16} \tau_{\pi^+}$):
- Little is experimentally known about the elastic form factors.
- Lattice-regularised QCD results are usually used as a guide.

Lattice-regularised QCD produce $\Delta$-resonance masses that are very large:

<table>
<thead>
<tr>
<th>Approach</th>
<th>$m_\pi$</th>
<th>$m_\rho$</th>
<th>$m_\Delta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unquenched I</td>
<td>0.691</td>
<td>0.986</td>
<td>1.687</td>
</tr>
<tr>
<td>Unquenched II</td>
<td>0.509</td>
<td>0.899</td>
<td>1.559</td>
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<tr>
<td>Unquenched III</td>
<td>0.384</td>
<td>0.848</td>
<td>1.395</td>
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<tr>
<td>Hybrid</td>
<td>0.353</td>
<td>0.959</td>
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<tr>
<td>Quenched I</td>
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<tr>
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<td>0.835</td>
<td>1.425</td>
</tr>
<tr>
<td>Quenched III</td>
<td>0.411</td>
<td>0.817</td>
<td>1.382</td>
</tr>
</tbody>
</table>

We artificially inflate the mass of the $\Delta$-baryon (right panels on the next...)
- Black solid line: DSEs + sophisticated interaction (with/without $m_\Delta = 1.8$ GeV)
- Blue dashed line: DSEs + contact interaction (with/without $m_\Delta = 1.8$ GeV)
$G_{E0}$ and $G_{M1}$ with(out) an inflated quark core mass

$G_{E0}$

$x = Q^2/m^2$

$G_{M1}$

$x = Q^2/m^2$
$G_{E2}$ and $G_{M3}$ with(out) an inflated quark core mass
Conclusions

The $\gamma^* N \rightarrow \Delta$ transition form factors

- Jones-Scadron $G_M^{*p}$:
  - $G_M^{*p}$ falls asymptotically at the same rate as $G_M^p$.
  - Compatible with isospin symmetry and pQCD predictions.
  - Data do not fall unexpectedly rapid once the kinematic relation between Jones-Scadron and Ash conventions is properly account for.

- $R_{EM}$ and $R_{SM}$:
  - Limits of pQCD, $R_{EM} \rightarrow 1$ and $R_{SM} \rightarrow$ constant, are apparent in our calculation but truly asymptotic $Q^2$ is required before the predictions are realized.
  - Strong diquark correlations within baryons produce a zero in the transition electric quadrupole at $Q^2 \sim 2 \text{ GeV}^2$.

The $\Delta$ elastic form factors

- The $\Delta$ elastic form factors are very sensitive to $m_\Delta$.

- Lattice-regularised QCD produce $\Delta$-resonance masses that are very large, the form factors obtained therewith should be interpreted carefully.