THE ELECTROMAGNETIC $\gamma^* N \to \Delta$ TRANSITION IN DYSON-SCHWINGER EQUATIONS

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The Δ baryon

Discovered more than 50 years ago

↓

By Fermi and collaborators

↓

In pion scattering off protons at the Chicago cyclotron (now Fermilab)

- E. Fermi *et al.*, Phys. Rev. **85**, 935 (1952).
- H. Anderson *et al.*, Phys. Rev. **85**, 936 (1952).

Citation: J. Beringer *et al.* (Particle Data Group), PR **D86**, 010001 (2012) (URL: http://pdg.lbl.gov)

$\Delta(1232)\ 3/2^{+}$

$I(J^P) = \frac{3}{2}(\frac{3}{2}^{+})$ Status: ****

- Mass of 1232 MeV and width of 120 MeV.
- Lightest baryon resonance ⇒ 300 MeV heavier than the nucleon.
- Almost an ideally elastic πN resonance ⇒ 99% of times decaying to $\Delta \to \pi N$.
- Only other decay channel: $\Delta \to \gamma N$ ⇒ less than 1% to the total decay width.

The Δ^+ and Δ^0 can be view, respectively, as isospin- and spin-flip excitations of the proton and neutron

Two ways in order to analyze the structure of the Δ-resonances

$\swarrow$ $\searrow$

π-mesons as a probe	photons as a probe
$\downarrow$	$\downarrow$
complex	relatively simple

BUT: $\mathcal{B}(\Delta \to \gamma N) \lesssim 1\%$

This became possible with the advent of intense, energetic electron-beam facilities

- Reliable data on the $\gamma^* p \to \Delta^+$ transition:
  ☞ Available on the entire domain $0 \leq Q^2 \leq 8\,\mathrm{GeV}^2$.
- Isospin symmetry implies $\gamma^* n \to \Delta^0$ is simply related with $\gamma^* p \to \Delta^+$.

$\gamma^* p \to \Delta^+$ *data has stimulated a great deal of theoretical analysis:*

- *Deformation of hadrons.*
- *The relevance of pQCD to processes involving moderate momentum transfers.*
- *The role that experiments on resonance electroproduction can play in exposing non-perturbative phenomena in QCD:*
  ☞ *The nature of confinement and Dynamical Chiral Symmetry Breaking.*

The electromagnetic current

☞ *The electromagnetic current can be generally written as:*

$$J_{\mu\lambda}(K,Q) = \Lambda_+(P_f) R_{\lambda\alpha}(P_f) i\gamma_5 \Gamma_{\alpha\mu}(K,Q)\Lambda_+(P_i)$$

- Incoming nucleon momentum $\Rightarrow P_i^2 = -m_N^2$.
- Outgoing Δ momentum $\Rightarrow P_f^2 = -m_\Delta^2$.
- $Q = P_f - P_i$ and $K = (P_i + P_f)/2$.
- The on-shell structure is ensured by the N- and Δ-baryon projectors.

☞ *The composition of the 4-point function $\Gamma_{\alpha\mu}$ is determined by Poincaré covariance:*

Convenient to work with orthogonal momenta $\leftrightarrow$ Simplify its structure considerably

$\downarrow$

Not yet the case for K and Q $\leftrightarrow$ $\Delta(m_\Delta - m_N) \neq 0 \Rightarrow K \cdot Q \neq 0$

$\downarrow$

We take instead $\hat{K}_\mu^\perp = \mathbb{T}_{\mu\nu}^Q \hat{K}_\nu$ and $\hat{Q}$

☞ *Vertex decomposes in terms of three (Jones-Scadron) form factors*

$$\Gamma_{\alpha\mu} = \mathpzc{k}\left[\frac{\lambda_m}{2\lambda_+}(G_M^* - G_E^*)\gamma_5\varepsilon_{\alpha\mu\gamma\delta}\hat{K}_\gamma^\perp\hat{Q}_\delta - G_E^*\mathbb{T}_{\alpha\gamma}^Q\mathbb{T}_{\gamma\mu}^K - \frac{i\varsigma}{\lambda_m}G_C^*\hat{Q}_\alpha\hat{K}_\mu^\perp\right],$$

Magnetic dipole $\Rightarrow G_M^*$ | Electric quadrupole $\Rightarrow G_E^*$ | Coulomb quadrupole $\Rightarrow G_C^*$

- The Jones-Scadron form factors are:

$$G_M^* = 3(s_2 + s_1),$$
$$G_E^* = s_2 - s_1,$$
$$G_C^* = s_3.$$

$G^*_{M,\mathrm{Ash}}$ vs $G^{*p}_{M,\mathrm{J-S}}$

$$G^*_{M,\mathrm{Ash}} = G^*_{M,\mathrm{J-S}}\left(1 + \frac{Q^2}{(m_\Delta + m_N)^2}\right)^{-\frac{1}{2}}$$

- The scalars are obtained from the following Dirac traces and momentum contractions:

$$s_1 = n\frac{\sqrt{\varsigma(1+2d)}}{d-\varsigma}\mathbb{T}^K_{\mu\nu}\hat{K}^\perp_\lambda \mathrm{Tr}[\gamma_5 J_{\mu\lambda}\gamma_\nu],$$
$$s_2 = n\frac{\lambda_+}{\lambda_m}\mathbb{T}^K_{\mu\lambda}\mathrm{Tr}[\gamma_5 J_{\mu\lambda}],$$
$$s_3 = 3n\frac{\lambda_+}{\lambda_m}\frac{(1+2d)}{d-\varsigma}\hat{K}^\perp_\mu \hat{K}^\perp_\lambda \mathrm{Tr}[\gamma_5 J_{\mu\lambda}].$$

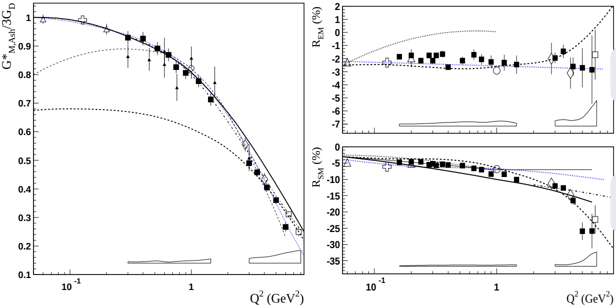
- We have used the following notation:

$$n = \frac{\sqrt{1-4d^2}}{4ik\lambda_m}, \quad \lambda_\pm = \frac{(m_\Delta \pm m_N)^2 + Q^2}{2(m_\Delta^2 + m_N^2)}, \quad \varsigma = \frac{Q^2}{2(m_\Delta^2 + m_N^2)},$$
$$d = \frac{m_\Delta^2 - m_N^2}{2(m_\Delta^2 + m_N^2)}, \quad \lambda_m = \sqrt{\lambda_+\lambda_-}, \quad k = \sqrt{\frac{3}{2}}\left(1 + \frac{m_\Delta}{m_N}\right).$$

G. Eichman et al., Phys. Rev. D **85**, *093004 (2012).*

Experimental results and theoretical expectations

I.G. Aznauryan and V.D. Burkert Prog. Part. Nucl Phys. **67**, 1-54 (2012)



☞ *The R_{EM} ratio is measured to be minus a few percent.*

☞ *The R_{SM} ratio does not seem to settle to a constant at large Q^2.*

pQCD predictions

For $Q^2 \to \infty$

- $G_M^* \to 1/Q^4$.
- $R_{EM} \to +100\%$.
- $R_{SM} \to \text{constant}$.

CQM predictions

Without quark orbital angular momentum:

- $R_{EM} \to 0$.
- $R_{SM} \to 0$.

SU(6) predictions

$$\langle p|\mu|\Delta^+\rangle = \langle n|\mu|\Delta^0\rangle$$

$$\langle p|\mu|\Delta^+\rangle = -\sqrt{2}\,\langle n|\mu|n\rangle$$

Data do not support these predictions

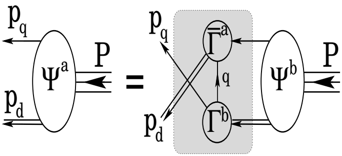
↓

Our aim: try to understand this longstanding puzzle

The attractive nature of quark-antiquark correlations in a color-singlet meson is also attractive for $\bar{3}_c$ quark-quark correlations within a color-singlet baryon

☞ Diquark correlations:

- A dynamical prediction of Faddeev equation studies.
- Non-pointlike color-antitriplet.
- Fully interacting.
- Empirical evidence in support of diquarks.



Diquark composition of the nucleon and Δ

Positive parity states

↙ pseudoscalar and vector diquarks ↓ Ignored, wrong parity, larger mass-scales

↘ scalar and axial-vector diquarks ↓ Dominant, right parity, shorter mass-scales → $N \Rightarrow 0^+, 1^+$ diquarks; $\Delta \Rightarrow$ only 1^+ diquark

Electromagnetic current description in the quark-diquark picture

To compute the electromagnetic properties of the $\gamma^* N\Delta$ reaction in a given framework, one must specify how the photon couples to its constituents.

☞ There are six contributions to the current.

The picture shows the one-loop diagrams

1. Coupling of the photon to the dressed quark.
2. Coupling of the photon to the dressed diquark:
   - ➪ Elastic transition.
   - ➪ Induced transition.

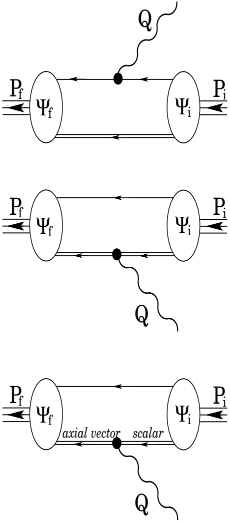
scalar diquark correlations are absent from the Δ-resonance

↓

Only axial-vector diquark correlations contribute in the top and middle diagrams

☞ Each diagram can be expressed like the electromagnetic current:

$$\Gamma_{\mu\lambda} = \Lambda_+(P_f) R_{\lambda\alpha}(P_f) \mathcal{J}_{\mu\alpha}(K, Q) \Lambda_+(P_i)$$



Elastic form factor of the proton in the quark-diquark picture

☞ Each diagram can be expressed in a similar way:

$$\Gamma_\mu = \Lambda_+(P_f)\mathcal{J}_\mu(K,Q)\Lambda_+(P_i)$$

Photon coupling directly to a dressed-quark with the diquark acting as a bystander

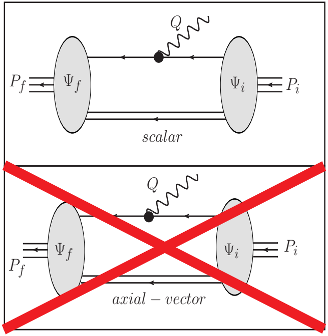
- Initial state and final state: Proton
  - Two axial-vector diquark isospin states:

    $(I, I_z) = (1,1) \rightarrow$ flavor content: $\{uu\}$

    $(I, I_z) = (1,0) \rightarrow$ flavor content: $\{ud\}$
  - In the isospin limit, they appear with relative weighting: $(-\sqrt{2/3}) : (\sqrt{1/3})$
- Therefore

$$\mathcal{J}_\mu^{\text{scalar}} = \sqrt{\frac{1}{3}}\sqrt{\frac{1}{3}}\, e_u\, I_\mu^{\{ud\}} \neq 0$$

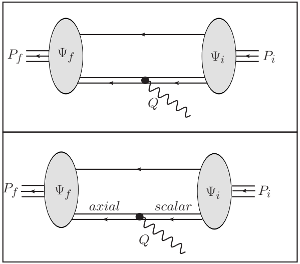
$$\mathcal{J}_\mu^{\text{axial}} = \sqrt{\frac{2}{3}}\sqrt{\frac{2}{3}}\, e_d\, I_\mu^{\{uu\}} + \sqrt{\frac{1}{3}}\sqrt{\frac{1}{3}}\, e_u\, I_\mu^{\{ud\}} = 0$$

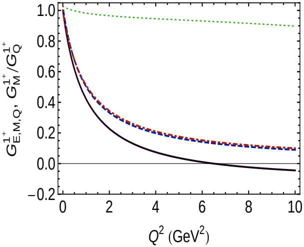


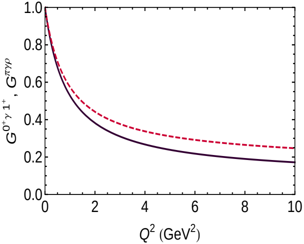
Hard contributions appear in the microscopic description of the elastic form factor of the proton

Remaining diagrams: Photon interacting with diquarks

H.L.L. Roberts *et al.* Phys. Rev. C **83**, 065206 (2011)







Composite object

↓

Electromagnetic radius is nonzero ($r_{qq} \gtrsim r_\pi$)

↓

Softer contribution to the form factors

Soft contributions appear in the microscopic description of the elastic form factor of the proton

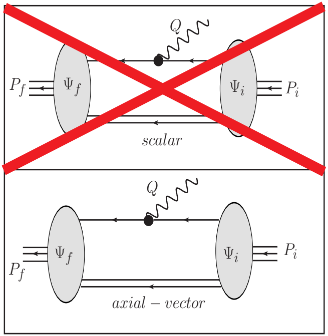
☞ Diagrams in which the photon interact with diquarks appear

Photon coupling directly to a dressed-quark with the diquark acting as a bystander

- Initial state: Proton
  - Two axial-vector diquark isospin states:

$$(I, I_z) = (1,1) \rightarrow \text{flavor content: } \{uu\}$$
$$(I, I_z) = (1,0) \rightarrow \text{flavor content: } \{ud\}$$

  - In the isospin limit, they appear with relative weighting: $(-\sqrt{2/3}) : (\sqrt{1/3})$



- Final state: Δ^+
  - Same isospin states of axial-vector diquark.
  - Different weighting due to $I_\Delta = 3/2$: $(\sqrt{1/3}) : (\sqrt{2/3})$

- Therefore

$$\mathcal{J}_{\mu\alpha}^{1,\text{axial}} = -\sqrt{\frac{2}{3}}\sqrt{\frac{1}{3}}\, e_d\, I_{\mu\alpha}^{1\{uu\}} + \sqrt{\frac{1}{3}}\sqrt{\frac{2}{3}}\, e_u\, I_{\mu\alpha}^{1\{ud\}} = \frac{\sqrt{2}}{3}\, I_{\mu\alpha}^{1\{qq\}}(K,Q)$$

Soft and still hard contributions appear in the microscopic description of the $\gamma^* N\Delta$ electromagnetic reaction

G_M^p vs G_M^{*p}

☞ *Similar contributions in both cases:*

G_M^{*p} *should fall asymptotically at the same rate as* G_M^p.

☞ *By isospin considerations:*

G_M^{*n} *should fall asymptotically at the same rate as* G_M^{*p}.

☞ *Hold SU(6):*

$$\langle p|\mu|\Delta^+\rangle \propto \langle n|\mu|\Delta^0\rangle \propto \langle p|\mu|p\rangle \,.$$

Symmetry preserving Dyson-Schwinger equation treatment of a vector × vector contact interaction

- Gluon propagator: Contact interaction.

$$g^2 D_{\mu\nu}(p-q) = \delta_{\mu\nu}\frac{4\pi\alpha_{\rm IR}}{m_{\rm G}^2}$$

- Truncation scheme: Rainbow-ladder.

$$\Gamma^a_\nu(q,p) = (\lambda^a/2)\gamma_\nu$$

- Fermion propagator: Gap equation.

$$S^{-1}(p) = i\gamma\cdot p + m + \Sigma(p)$$
$$= i\gamma\cdot p + M$$

  - $M \sim 0.4\,\mathrm{GeV} = \mathrm{constant}$.
  - Implies momentum independent Bethe-Salpeter and Faddeev amplitudes.

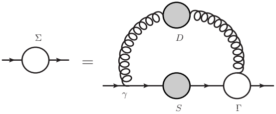
- Baryons: Faddeev equation.

$$m_N = 1.14\,\mathrm{GeV} \quad m_\Delta = 1.39\,\mathrm{GeV}$$

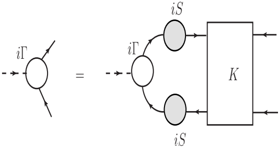
(masses reduced by pion-cloud effects)

- Ward-Green-Takahashi identities: Axial-vector and vector.

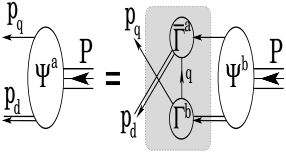
Gap equation



Bethe-Salpeter equation



Faddeev equation



Series of papers establishes strengths and limitations

Used judiciously, produces results indistinguishable from most sophisticated Rainbow-ladder interactions

- Spectrum of hadrons with strangeness
  Chen Chen, L. Chang, C.D. Roberts, Shaolong Wang and D.J. Wilson
  Few Body Syst. **53** 293-326 (2012). arXiv:1204.2553 [nucl-th]
- Nucleon and Roper electromagnetic elastic and transition form factors
  D.J. Wilson, I.C. Cloët, L. Chang and C.D. Roberts
  Phys. Rev. C **85**, 025205 (2012). arXiv:1112.2212 [nucl-th]
- π- and ρ-mesons, and their diquark partners, from a contact interaction
  H.L.L. Roberts, A. Bashir, L.X. Gutierrez-Guerrero, C.D. Roberts and D.J. Wilson
  Phys. Rev. C **83**, 065206 (2011). arXiv:1102.4376 [nucl-th]
- Masses of ground and excited-state hadrons
  H.L.L. Roberts, L. Chang, I.C. Cloët and C.D. Roberts
  Few Body Syst. **51**, 1-25 (2011). arXiv:1101.4244 [nucl-th]
- Abelian anomaly and neutral pion production
  H.L.L. Roberts, C.D. Roberts, A. Bashir, L.X. Gutierrez-Guerrero and P.C. Tandy
  Phys. Rev. C **82**, 065202 (2010). arXiv:1009.0067 [nucl-th]
- Pion form factor from a contact interaction
  L.X. Gutierrez-Guerrero, A. Bashir, I.C. Cloët and C.D. Roberts
  Phys. Rev. C **81**, 065202 (2010). arXiv:1002.1968 [nucl-th]

Weakness of contact-interaction

☞ Truncation which produces Faddeev amplitudes that are independent of relative momentum:

- Underestimates the quark orbital angular momentum content of the bound-state.
- Suppresses the two-loop diagrams.

	ind.-p DSE kernels	dep.-p DSE kernels
axial-diquark(Δ)-axial-diquark(p)	0.85	0.96
axial-diquark(Δ)-scalar-diquark(p)	0.18	1.27

Two sets of results

1. Original result.
2. Improved version:
   - Rescale the axial(Δ)-scalar(p) diagram

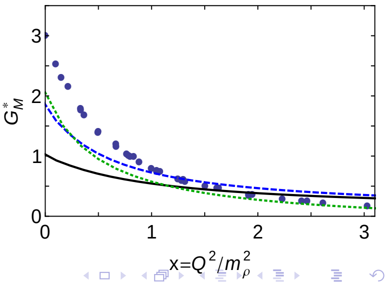
$$1 + \frac{g_{\rm as/aa}}{1 + Q^2/m_\rho^2}$$

$$\boxed{\text{axial}(\Delta)\text{-scalar}(p) = \text{axial}(\Delta)\text{-axial}(p)}$$

   - Incorporate dressed quark-anomalous magnetic moment

     ☞ Consequence of the DCSB.

Coupled-channel prediction of the dressed quark core contribution



Q^2-behaviour of $G^{*p}_{M,\mathrm{Ash}}$

G^{*p}_M fall asymptotically at the same rate as $G^p_M \propto 1/Q^4$

Historically
experimental data has been presented
in the Ash *et al.* convention

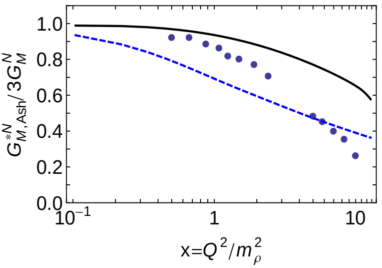
↓

$G^{*}_{M,\mathrm{Ash}}$ vs $G^{*p}_{M,\mathrm{J-S}}$

$$G^{*}_{M,\mathrm{Ash}} = G^{*}_{M,\mathrm{J-S}} \left(1 + \frac{Q^2}{(m_\Delta + m_N)^2}\right)^{-\frac{1}{2}}$$

↓

A factor $1/Q$ of difference



Meson-cloud effects

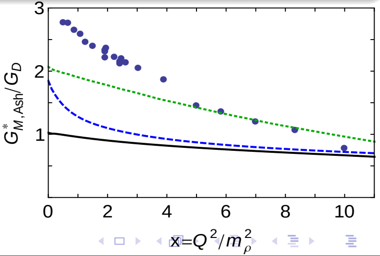
Provide more than 30% for $Q^2 \lesssim 2m^2_\rho$

↓

These contributions are very soft

↓

They disappear rapidly



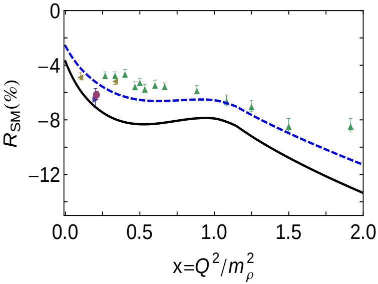
R_{EM} and R_{SM} ratios

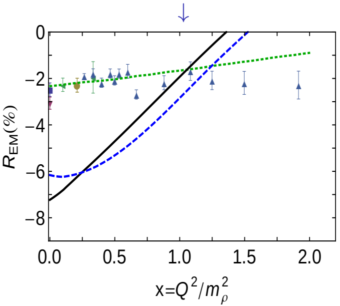
☞ Measures:

- Deformation of the hadrons involved.
- The influence on the structure of the transition current.

☞ R_{EM} is more sensitive to quark orbital angular momentum:

- The true amount of which is predicted poorly by the contact interaction.

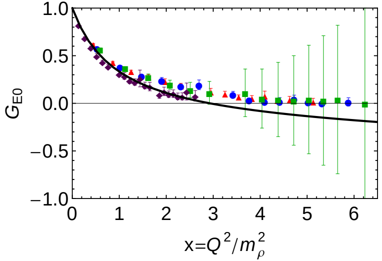


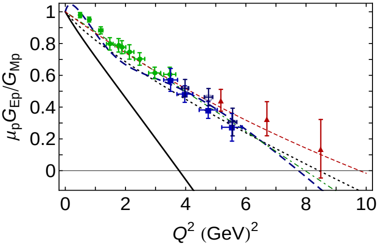


Contact interaction produces correlations between dressed-quarks within baryon and associated features in the current that are comparable in size with those observed empirically

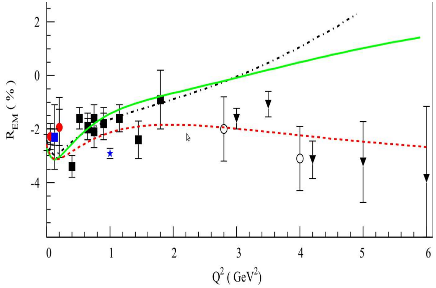
Contact interection predicts zero crossing for the electric form factors of the hadrons involved

- ☞ *The existence of a zero is independent of the interaction*
- ☞ *The location of the zero depends on the interaction.*





Experimental data and dynamical models do not rule out the possibility of a zero crossing in the transition electric form factor.

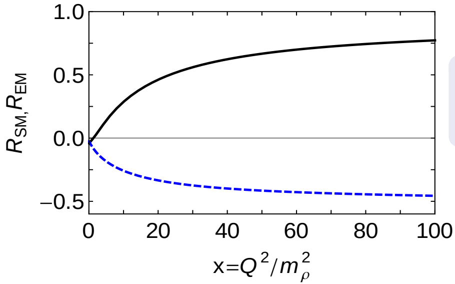


Vladimir Pascalutsa Phys. Rep. 437, 125 (2007).

Large Q^2-behaviour of the quadrupole ratios

Helicity conservation arguments in pQCD should apply equally to an internally-consistent symmetry-preserving treatment of a contact interaction

$$R_{EM} \overset{Q^2\to\infty}{=} 1, \quad R_{SM} \overset{Q^2\to\infty}{=} \text{constant}$$



Take in mind that the asymptotic power-law dependence of our computed form factors is harder that in QCD.

Observations:

- Truly asymptotic Q^2 is required before predictions are realized.
- $G_E^*(Q^2)$ posses a zero at an empirical accessible momentum and thereafter $R_{EM} \to 1$.
- $R_{SM} \to \text{constant}$. The curve we display contains the $ln^2 Q^2$-growth expected in QCD but it is not a prominent feature.

Epilogue:
We have computed the $\gamma^* N \to \Delta$ transition form factors using a Poincaré-covariant, symmetry-preserving treatment of a vector $\times$ vector contact interaction.

- Jones-Scadron G_M^{*p}:

☞ G_M^{*p} fall asymptotically at the same rate as G_M^p.

☞ Compatible with isospin symmetry and pQCD predictions.

☞ Data do not fall unexpectedly rapid once the kinematic relation between Jones-Scadron and Ash form factors is properly account for.

- G_E^{*p}

☞ The presence of strong diquark correlations within baryons predicts zero crossings for the electric form factors of the baryons involve in the $\gamma^* N \to \Delta$ transition.

☞ This implies that there should be a zero in the transition electric form factor.

☞ Experimental data and dynamical models do not rule out this possibility for the transition electric form factor.

- R_{EM} and R_{SM}:

☞ Contact interaction produces correlations between dressed-quarks within Faddeev wave functions and related features in the current that are comparable in size with those observed empirically.

☞ Limits of pQCD, $R_{EM} \to 1$ and $R_{SM} \to \text{constant}$, are apparent in our calculation but truly asymptotic Q^2 is required before the predictions are realized.