STUDY OF ELECTROMAGNETIC FORM FACTORS WITH DYSON-SCHWINGER EQUATIONS

Jorge Segovia, Ian C. Cloet and Craig D. Roberts
Argonne National Laboratory

Chen Chen and Shaolong Wang
University of Science and Technology of China

XXVI Midwest Theory Get-Together
Argonne National Laboratory
September 6-7, 2013

Quantum Chromodynamics

☞ The non-Abelian gauge quantum field theory that describes quark and gluon physics.

- Successful at high energies → perturbative calculations are allowed.
- Nonperturbative regime → hadron properties → remains to be understood.

Emergent phenomena

☞ Neither of these phenomena is apparent in QCD's Lagrangian yet! they are the dominant determining characteristics of hadrons.

- Quark and gluon confinement.
- Dynamical chiral symmetry breaking (DCSB).

Dyson-Schwinger equations (DSEs)

☞ Provides a nonperturbative tool for the study of continuum strong QCD.

- Allows the study of the interaction between light quarks in the whole range of momenta.
- Analysis of the infrared behaviour of the strong coupling constant (β-function) is crucial to disentangle confinement and DCSB.
- DSEs connect β-function to experimental observables: Hadron mass spectrum, elastic and transition form factors ...

Study of baryon electromagnetic form factors

- A central goal of (the DOE Office of) Nuclear Physics is to understand the structure and properties of protons and neutrons, and ultimately atomic nuclei, in terms of the quarks and gluons of QCD.
- The excitation of nucleon resonances in electromagnetic interactions has long been recognize as an important source of information for understanding strong interactions.

We have one of the most beautiful and expensive machines around the world: Jefferson Lab

☞ The most accurate results have been obtained for the electroexcitation amplitudes of the four lowest excited states:

- $8.0\,\mathrm{GeV}^2$ for $\Delta(1232)P_{33}$ and $N(1535)S_{11}$.
- $4.5\,\mathrm{GeV}^2$ for $N(1440)P_{11}$ and $N(1520)D_{13}$.

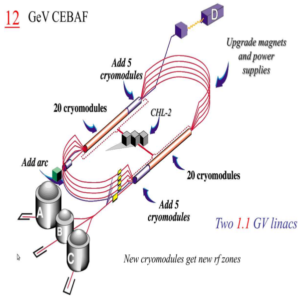
Upgrade of JLab up to $12\,\mathrm{GeV}$

☞ Upgrade underway.

☞ Construction cost $310-million.

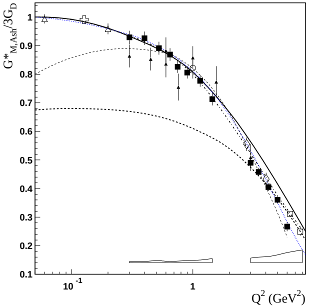
☞ New generation experiments in 2015.

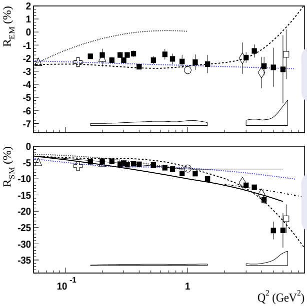
☞ In particular, a dedicated experiment will aim to extract the N^* electrocouplings at photon virtualities Q^2 ever achieved so far.



☞ Experimental data has stimulated a great deal of theoretical analysis about:

- Deformation of hadrons.
- The relevance of pQCD to processes involving moderate momentum transfers.
- The role that experiments on resonance electroproduction can play in exposing non-perturbative phenomena in QCD.





☞ *The R_{EM} ratio is measured to be minus a few percent.*

☞ *The R_{SM} ratio does not seem to settle to a constant at large Q^2.*

pQCD predictions

For $Q^2 \to \infty$

- $G_M^* \to 1/Q^4$.
- $R_{EM} \to +100\%$.
- $R_{SM} \to$ constant.

CQM predictions

Without quark orbital angular momentum:

- $R_{EM} \to 0$.
- $R_{SM} \to 0$.

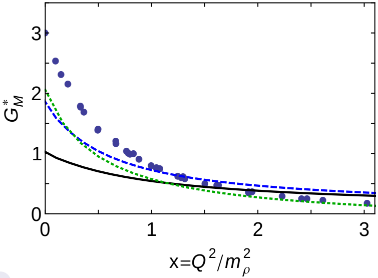
SU(6) predictions

$$\langle p|\mu|\Delta^+\rangle = \langle n|\mu|\Delta^0\rangle$$

$$\langle p|\mu|\Delta^+\rangle = -\sqrt{2}\,\langle n|\mu|n\rangle$$

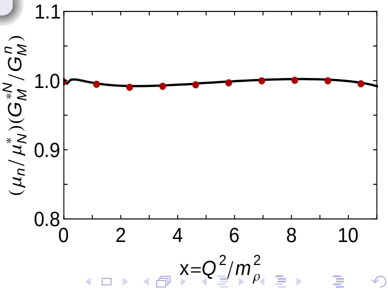
G^*_M vs. Experimental data and dynamical models

- Both computed curves are consistent with data for $Q^2 \gtrsim 2m_\rho^2$.
- They are in marked disagreement at infrared momenta.
- Similarity between our improved version and dressed-quark-core result determined by EBAC.
- The discrepancy results from the omission of meson-cloud effects.



Transition vs. elastic magnetic form factors

- The fall-off rate of $G^*_M(Q^2)$ in the $\gamma^* p \to \Delta^+$ transition must much that of $G_M(Q^2)$.
- With isospin symmetry, $\langle p|\mu|\Delta^+\rangle = -\langle n|\mu|\Delta^0\rangle$ is valid, so same is true of the $\gamma^* n \to \Delta^0$ magnetic form factor.
- These are statements about the dressed quark core contributions $\to$ Outside the domain of meson-cloud effects, $Q^2 \gtrsim 2\,\mathrm{GeV}^2$



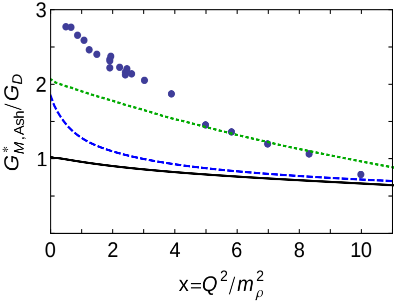
☞ Presentations of experimental data typically use the Ash convention.

- $G^*_{M,\mathrm{Ash}}(Q^2)$ falls faster than a dipole.
- Many have viewed this as a conundrum.
- There is no sound reason to expect:

$$G^*_{M,\mathrm{Ash}}/G^n_M \sim \text{constant}$$

- Instead, the Jones-Scadron form factor should exhibit:

$$G^*_{M,\mathrm{J-S}}/G^n_M \sim \text{constant}$$



Two main reasons

Meson-cloud effects

↓

Provide more than 30% for $Q^2 \lesssim 2m^2_\rho$

↓

These contributions are very soft

↓

They disappear rapidly

$G^*_{M,\mathrm{Ash}}$ vs. $G^*_{M,\mathrm{J-S}}$

$$G^*_{M,\mathrm{Ash}} = G^*_{M,\mathrm{J-S}} \left(1 + \frac{Q^2}{(m_\Delta + m_N)^2}\right)^{-\frac{1}{2}}$$

↓

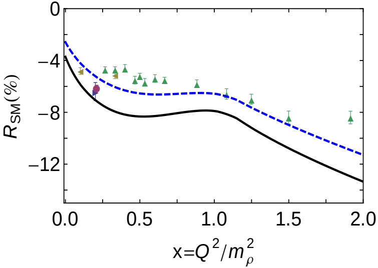
A factor $1/Q$ of difference

↓

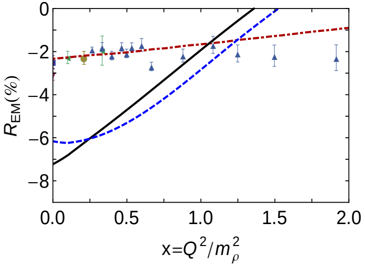
Provides material damping for $Q^2 \gtrsim 4m^2_\rho$

Electric and coulomb quadrupoles

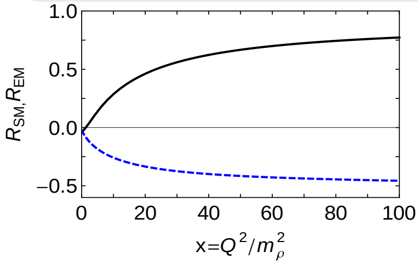
☞ R_{SM}: Good description of the rapid fall at large momentum transfer.



☞ R_{EM}: A particularly sensitive measure of orbital angular momentum correlations.



Large Q^2-behaviour of the quadrupole ratios



Observations:

- Truly asymptotic Q^2 is required before predictions are realized.
- $G_E^*(Q^2)$ posses a zero at an empirical accessible momentum and thereafter $R_{EM} \to 1$.
- $R_{SM} \to \mathrm{constant}$. The curve we display contains the $ln^2 Q^2$-growth expected in QCD.

- Hadron Physics, Dyson-Schwinger equations and the study of baryon electromagnetic form factors:

  ☞ Confinement and DCSB have a deep impact on hadron physics.

  ☞ DSEs is a suitable approach because connects naturally QCD features with hadron properties.

  ☞ Baryon form factors are an important source of information for understanding strong interactions.

The $\gamma^* N \to \Delta$ reaction

- Jones-Scadron G_M^{*p}:

  ☞ G_M^{*p} falls asymptotically at the same rate as G_M^p.

  ☞ Compatible with isospin symmetry and pQCD.

  ☞ Data do not fall unexpectedly rapid:

  Jones-Scadron vs. Ash conventions + Pion Cloud

- R_{EM} and R_{SM}:

  ☞ Different from zero → Deformation of hadrons.

  ☞ Limits of pQCD, $R_{EM} \to 1$ and $R_{SM} \to \text{constant}$, are apparent in our calculation.

Outlook

☞ *Verify claims using a more sophisticated interaction.*

☞ *Compute transition form factors for $N \to N(1535)S_{11}$.*

☞ *Projects are essential in paving the way for JLab to chart the infrared behaviour of QCD.*

BACKUP SLIDES

The electromagnetic current

☞ *The electromagnetic current can be generally written as:*

$$J_{\mu\lambda}(K,Q) = \Lambda_+(P_f) R_{\lambda\alpha}(P_f) i\gamma_5 \Gamma_{\alpha\mu}(K,Q) \Lambda_+(P_i)$$

- Incoming nucleon momentum $\Rightarrow P_i^2 = -m_N^2$.
- Outgoing Δ momentum $\Rightarrow P_f^2 = -m_\Delta^2$.
- $Q = P_f - P_i$ and $K = (P_i + P_f)/2$.
- The on-shell structure is ensured by the N- and Δ-baryon projectors.

☞ *The composition of the 4-point function $\Gamma_{\alpha\mu}$ is determined by Poincaré covariance:*

Convenient to work with orthogonal momenta $\leftrightarrow$ Simplify its structure considerably

$\downarrow$

Not yet the case for K and Q $\leftrightarrow$ $\Delta(m_\Delta - m_N) \neq 0 \Rightarrow K \cdot Q \neq 0$

$\downarrow$

We take instead $\hat{K}_\mu^\perp = \mathbb{T}_{\mu\nu}^Q \hat{K}_\nu$ and $\hat{Q}$

☞ *Vertex decomposes in terms of three (Jones-Scadron) form factors*

$$\Gamma_{\alpha\mu} = \mathcal{k}\left[\frac{\lambda_m}{2\lambda_+}(G_M^* - G_E^*)\gamma_5 \varepsilon_{\alpha\mu\gamma\delta} \hat{K}_\gamma^\perp \hat{Q}_\delta - G_E^* \mathbb{T}_{\alpha\gamma}^Q \mathbb{T}_{\gamma\mu}^K - \frac{i\varsigma}{\lambda_m} G_C^* \hat{Q}_\alpha \hat{K}_\mu^\perp\right],$$

Magnetic dipole $\Rightarrow G_M^*$ | Electric quadrupole $\Rightarrow G_E^*$ | Coulomb quadrupole $\Rightarrow G_C^*$

- The Jones-Scadron form factors are:

$$G_M^* = 3(s_2 + s_1),$$
$$G_E^* = s_2 - s_1,$$
$$G_C^* = s_3.$$

$G^*_{M,\mathrm{Ash}}$ vs $G^{*p}_{M,\mathrm{J-S}}$

$$G^*_{M,\mathrm{Ash}} = G^*_{M,\mathrm{J-S}}\left(1 + \frac{Q^2}{(m_\Delta + m_N)^2}\right)^{-\frac{1}{2}}$$

- The scalars are obtained from the following Dirac traces and momentum contractions:

$$s_1 = n\frac{\sqrt{\varsigma(1+2d)}}{d-\varsigma}\mathbb{T}^K_{\mu\nu}\hat{K}^\perp_\lambda \mathrm{Tr}[\gamma_5 J_{\mu\lambda}\gamma_\nu],$$
$$s_2 = n\frac{\lambda_+}{\lambda_m}\mathbb{T}^K_{\mu\lambda}\mathrm{Tr}[\gamma_5 J_{\mu\lambda}],$$
$$s_3 = 3n\frac{\lambda_+}{\lambda_m}\frac{(1+2d)}{d-\varsigma}\hat{K}^\perp_\mu \hat{K}^\perp_\lambda \mathrm{Tr}[\gamma_5 J_{\mu\lambda}].$$

- We have used the following notation:

$$n = \frac{\sqrt{1-4d^2}}{4i\,k\,\lambda_m}, \quad \lambda_\pm = \frac{(m_\Delta \pm m_N)^2 + Q^2}{2(m_\Delta^2 + m_N^2)}, \quad \varsigma = \frac{Q^2}{2(m_\Delta^2 + m_N^2)},$$
$$d = \frac{m_\Delta^2 - m_N^2}{2(m_\Delta^2 + m_N^2)}, \quad \lambda_m = \sqrt{\lambda_+\lambda_-}, \quad k = \sqrt{\frac{3}{2}}\left(1 + \frac{m_\Delta}{m_N}\right).$$

G. Eichman et al., Phys. Rev. D **85**, *093004 (2012).*

Symmetry preserving Dyson-Schwinger equation treatment of a vector $\times$ vector contact interaction

- **Gluon propagator:** Contact interaction.

$$g^2 D_{\mu\nu}(p-q) = \delta_{\mu\nu} \frac{4\pi\alpha_{\rm IR}}{m_{\rm G}^2}$$

- **Truncation scheme:** Rainbow-ladder.

$$\Gamma_\nu^a(q,p) = (\lambda^a/2)\gamma_\nu$$

- **Fermion propagator:** Gap equation.

$$S^{-1}(p) = i\gamma\cdot p + m + \Sigma(p) = i\gamma\cdot p + M$$

  - $M \sim 0.4\,\mathrm{GeV} = \mathrm{constant}$.
  - Implies momentum independent Bethe-Salpeter and Faddeev amplitudes.

- **Baryons:** Faddeev equation.

$$m_N = 1.14\,\mathrm{GeV} \quad m_\Delta = 1.39\,\mathrm{GeV}$$

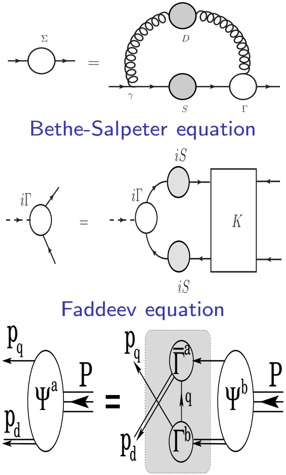
(masses reduced by pion-cloud effects)

- **Ward-Green-Takahashi identities:** Axial-vector and vector.

Gap equation

Bethe-Salpeter equation

Faddeev equation



Series of papers establishes strengths and limitations

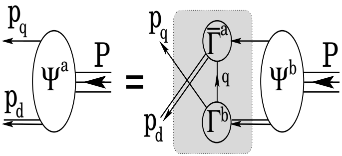
Used judiciously, produces results indistinguishable from most-sophisticated Rainbow-ladder interactions

- Spectrum of hadrons with strangeness
  Chen Chen, L. Chang, C.D. Roberts, Shaolong Wang and D.J. Wilson
  Few Body Syst. **53** 293-326 (2012). arXiv:1204.2553 [nucl-th]
- Nucleon and Roper electromagnetic elastic and transition form factors
  D.J. Wilson, I.C. Cloët, L. Chang and C.D. Roberts
  Phys. Rev. C **85**, 025205 (2012). arXiv:1112.2212 [nucl-th]
- π- and ρ-mesons, and their diquark partners, from a contact interaction
  H.L.L. Roberts, A. Bashir, L.X. Gutierrez-Guerrero, C.D. Roberts and D.J. Wilson
  Phys. Rev. C **83**, 065206 (2011). arXiv:1102.4376 [nucl-th]
- Masses of ground and excited-state hadrons
  H.L.L. Roberts, L. Chang, I.C. Cloët and C.D. Roberts
  Few Body Syst. **51**, 1-25 (2011). arXiv:1101.4244 [nucl-th]
- Abelian anomaly and neutral pion production
  H.L.L. Roberts, C.D. Roberts, A. Bashir, L.X. Gutierrez-Guerrero and P.C. Tandy
  Phys. Rev. C **82**, 065202 (2010). arXiv:1009.0067 [nucl-th]
- Pion form factor from a contact interaction
  L.X. Gutierrez-Guerrero, A. Bashir, I.C. Cloët and C.D. Roberts
  Phys. Rev. C **81**, 065202 (2010). arXiv:1002.1968 [nucl-th]

The attractive nature of quark-antiquark correlations in a color-singlet meson is also attractive for $\bar{3}_c$ quark-quark correlations within a color-singlet baryon

☞ Diquark correlations:

- A dynamical prediction of Faddeev equation studies.
- Non-pointlike color-antitriplet.
- Fully interacting.
- Empirical evidence in support of diquarks.



Diquark composition of the nucleon and Δ

Positive parity states

↙ pseudoscalar and vector diquarks ↓ Ignored, wrong parity, larger mass-scales

↘ scalar and axial-vector diquarks ↓ Dominant, right parity, shorter mass-scales → $N \Rightarrow 0^+, 1^+$ diquarks; $\Delta \Rightarrow$ only 1^+ diquark

Weakness of contact-interaction

☞ Truncation which produces Faddeev amplitudes that are independent of relative momentum:

- Underestimates the quark orbital angular momentum content of the bound-state.
- Suppresses the two-loop diagrams.

	ind.-p DSE kernels	dep.-p DSE kernels
axial-diquark(Δ)-axial-diquark(p)	0.85	0.96
axial-diquark(Δ)-scalar-diquark(p)	0.18	1.27

Two sets of results

1. Original result.
2. Improved version:
   - Rescale the axial(Δ)-scalar(p) diagram

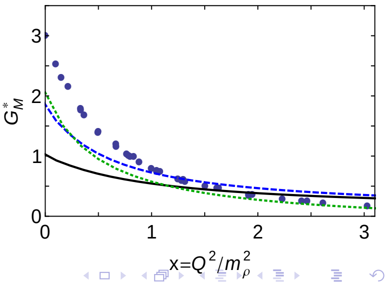
$$1 + \frac{g_{\mathrm{as/aa}}}{1 + Q^2/m_\rho^2}$$

$$\boxed{\text{axial}(\Delta)\text{-scalar}(p) = \text{axial}(\Delta)\text{-axial}(p)}$$

   - Incorporate dressed quark-anomalous magnetic moment

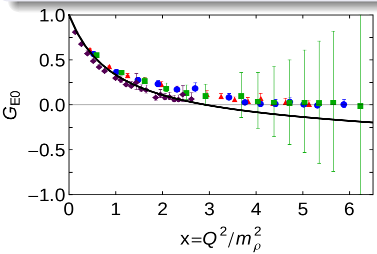
☞ Consequence of the DCSB.

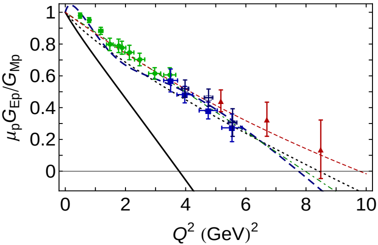
Coupled-channel prediction of the dressed quark core contribution



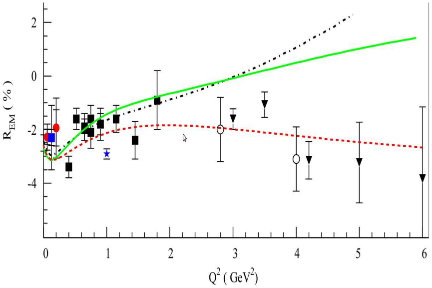
Contact interaction predicts zero crossing for the electric form factors of the hadrons involved

- ☞ *The existence of a zero is independent of the interaction*
- ☞ *The location of the zero depends on the interaction.*





Experimental data and dynamical models do not rule out the possibility of a zero crossing in the transition electric form factor.



Vladimir Pascalutsa Phys. Rep. 437, 125 (2007).