EXPLORING $N \to \Delta$ TRANSITION USING DYSON-SCHWINGER EQUATIONS

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Discovered more than 50 years ago

↓

By Fermi and collaborators

↓

In pion scattering off protons at the Chicago cyclotron (now Fermilab)

- E. Fermi *et al.*, Phys. Rev. **85**, 935 (1952).
- H. Anderson *et al.*, Phys. Rev. **85**, 936 (1952).

Citation: J. Beringer *et al.* (Particle Data Group), PR **D86**, 010001 (2012) (URL: http://pdg.lbl.gov)

Δ(1232) 3/2$^+$

$I(J^P) = \frac{3}{2}(\frac{3}{2}^+)$ Status: * * * *

- Mass of 1232 MeV and width of 120 MeV.
- Lightest baryon resonance $\Rightarrow$ 300 MeV heavier than the nucleon.
- Almost an ideally elastic πN resonance $\Rightarrow$ 99% of times decaying to $\Delta \to \pi N$.
- Only other decay channel: $\Delta \to \gamma N \Rightarrow$ less than 1% to the total decay width.

The Δ^+ and Δ^0 can be view, respectively, as isospin- and spin-flip excitations of the proton and neutron

Two ways in order to analyze the structure of the Δ-resonances

$\swarrow$ $\searrow$

π-mesons as a probe	photons as a probe
$\downarrow$	$\downarrow$
complex	relatively simple

BUT: $\mathcal{B}(\Delta \to \gamma N) \lesssim 1\%$

This became possible with the advent of intense, energetic electron-beam facilities

- Reliable data on the $\gamma^* p \to \Delta^+$ transition:
  ☞ Available on the entire domain $0 \leq Q^2 \leq 8\,\mathrm{GeV}^2$.
- Isospin symmetry implies $\gamma^* n \to \Delta^0$ is simply related with $\gamma^* p \to \Delta^+$.

$\gamma^* p \to \Delta^+$ *data has stimulated a great deal of theoretical analysis:*

- *Deformation of hadrons.*
- *The relevance of pQCD to processes involving moderate momentum transfers.*
- *The role that experiments on resonance electroproduction can play in exposing non-perturbative phenomena in QCD:*
  ☞ *The nature of confinement and Dynamical Chiral Symmetry Breaking.*

☞ *The electromagnetic current can be generally written as:*

$$J_{\mu\lambda}(K,Q) = \Lambda_+(P_f) R_{\lambda\alpha}(P_f) i\gamma_5 \Gamma_{\alpha\mu}(K,Q) \Lambda_+(P_i)$$

- Incoming nucleon momentum $\Rightarrow P_i^2 = -m_N^2$.
- Outgoing Δ momentum $\Rightarrow P_f^2 = -m_\Delta^2$.
- $Q = P_f - P_i$ and $K = (P_i + P_f)/2$.
- The on-shell structure is ensured by the N- and Δ-baryon projectors.

☞ *The composition of the 4-point function $\Gamma_{\alpha\mu}$ is determined by Poincaré covariance:*

Convenient to work with orthogonal momenta $\leftrightarrow$ Simplify its structure considerably

$\downarrow$

Not yet the case for K and Q $\leftrightarrow$ $\Delta(m_\Delta - m_N) \neq 0 \Rightarrow K \cdot Q \neq 0$

$\downarrow$

We take instead $\hat{K}_\mu^\perp = \mathbb{T}_{\mu\nu}^Q \hat{K}_\nu$ and $\hat{Q}$

☞ *Vertex decomposes in terms of three (Jones-Scadron) form factors*

$$\Gamma_{\alpha\mu} = k \left[\frac{\lambda_m}{2\lambda_+} (G_M^* - G_E^*) \gamma_5 \varepsilon_{\alpha\mu\gamma\delta} \hat{K}_\gamma^\perp \hat{Q}_\delta - G_E^* \mathbb{T}_{\alpha\gamma}^Q \mathbb{T}_{\gamma\mu}^K - \frac{i\varsigma}{\lambda_m} G_C^* \hat{Q}_\alpha \hat{K}_\mu^\perp \right],$$

Magnetic dipole $\Rightarrow G_M^*$ | Electric quadrupole $\Rightarrow G_E^*$ | Coulomb quadrupole $\Rightarrow G_C^*$

- The Jones-Scadron form factors are:

$$G_M^* = 3(s_2 + s_1),$$
$$G_E^* = s_2 - s_1,$$
$$G_C^* = s_3.$$

$G^*_{M,\mathrm{Ash}}$ vs $G^{*p}_{M,\mathrm{J-S}}$

$$G^*_{M,\mathrm{Ash}} = G^*_{M,\mathrm{J-S}} \left(1 + \frac{Q^2}{(m_\Delta + m_N)^2}\right)^{-\frac{1}{2}}$$

- The scalars are obtained from the following Dirac traces and momentum contractions:

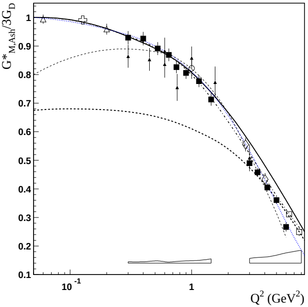
$$s_1 = n \frac{\sqrt{\varsigma(1+2d)}}{d - \varsigma} \mathbb{T}^K_{\mu\nu} \hat{K}^\perp_\lambda \mathrm{Tr}[\gamma_5 J_{\mu\lambda} \gamma_\nu],$$
$$s_2 = n \frac{\lambda_+}{\lambda_m} \mathbb{T}^K_{\mu\lambda} \mathrm{Tr}[\gamma_5 J_{\mu\lambda}],$$
$$s_3 = 3n \frac{\lambda_+}{\lambda_m} \frac{(1+2d)}{d - \varsigma} \hat{K}^\perp_\mu \hat{K}^\perp_\lambda \mathrm{Tr}[\gamma_5 J_{\mu\lambda}].$$

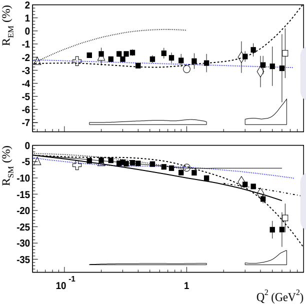
- We have used the following notation:

$$n = \frac{\sqrt{1-4d^2}}{4ik\lambda_m}, \quad \lambda_\pm = \frac{(m_\Delta \pm m_N)^2 + Q^2}{2(m_\Delta^2 + m_N^2)}, \quad \varsigma = \frac{Q^2}{2(m_\Delta^2 + m_N^2)},$$
$$d = \frac{m_\Delta^2 - m_N^2}{2(m_\Delta^2 + m_N^2)}, \quad \lambda_m = \sqrt{\lambda_+ \lambda_-}, \quad k = \sqrt{\frac{3}{2}}\left(1 + \frac{m_\Delta}{m_N}\right).$$

G. Eichman et al., Phys. Rev. D **85**, *093004 (2012).*

I.G. Aznauryan and V.D. Burkert Prog. Part. Nucl Phys. **67**, 1-54 (2012)





☞ *The R_{EM} ratio is measured to be minus a few percent.*

☞ *The R_{SM} ratio does not seem to settle to a constant at large Q^2.*

pQCD predictions

For $Q^2 \to \infty$

- $G_M^* \to 1/Q^4$.
- $R_{EM} \to +100\%$.
- $R_{SM} \to \text{constant}$.

CQM predictions

Without quark orbital angular momentum:

- $R_{EM} \to 0$.
- $R_{SM} \to 0$.

SU(6) predictions

$$\langle p|\mu|\Delta^+\rangle = \langle n|\mu|\Delta^0\rangle$$

$$\langle p|\mu|\Delta^+\rangle = -\sqrt{2}\,\langle n|\mu|n\rangle$$

Data do not support these predictions

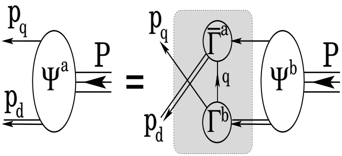
↓

Our aim: try to understand this longstanding puzzle

The attractive nature of quark-antiquark correlations in a color-singlet meson is also attractive for $\bar{3}_c$ quark-quark correlations within a color-singlet baryon

☞ Diquark correlations:

- A dynamical prediction of Faddeev equation studies.
- Non-pointlike color-antitriplet.
- Fully interacting.
- Empirical evidence in support of diquarks.



Diquark composition of the nucleon and Δ

Positive parity states

↙ pseudoscalar and vector diquarks ↓ Ignored, wrong parity, larger mass-scales

↘ scalar and axial-vector diquarks ↓ Dominant, right parity, shorter mass-scales → N ⇒ 0^+, 1^+ diquarks; Δ ⇒ only 1^+ diquark

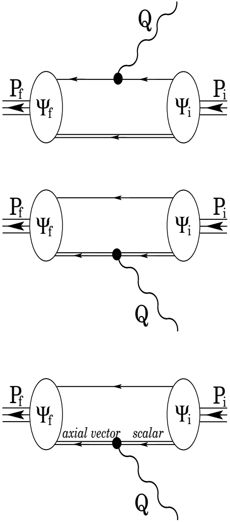
Electromagnetic current description in the quark-diquark picture

To compute the electromagnetic properties of the $\gamma^* N\Delta$ reaction in a given framework, one must specify how the photon couples to its constituents.

☞ There are six contributions to the current.

The picture shows the one-loop diagrams

1. Coupling of the photon to the dressed quark.
2. Coupling of the photon to the dressed diquark:
   - ➪ Elastic transition.
   - ➪ Induced transition.



scalar diquark correlations are absent from the Δ-resonance

↓

Only axial-vector diquark correlations contribute in the top and middle diagrams

☞ Each diagram can be expressed like the electromagnetic current:

$$\Gamma_{\mu\lambda} = \Lambda_+(P_f) R_{\lambda\alpha}(P_f) \mathcal{J}_{\mu\alpha}(K,Q) \Lambda_+(P_i)$$

☞ Each diagram can be expressed in a similar way:

$$\Gamma_\mu = \Lambda_+(P_f)\mathcal{J}_\mu(K,Q)\Lambda_+(P_i)$$

Photon coupling directly to a dressed-quark with the diquark acting as a bystander

- Initial state and final state: Proton
  - Two axial-vector diquark isospin states:

    $(I, I_z) = (1,1) \to$ flavor content: $\{uu\}$

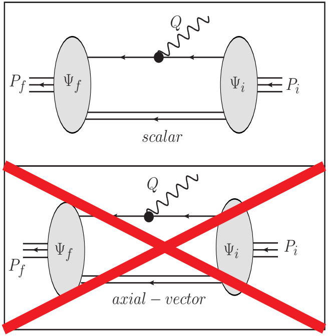
    $(I, I_z) = (1,0) \to$ flavor content: $\{ud\}$

  - In the isospin limit, they appear with relative weighting: $(-\sqrt{2/3}) : (\sqrt{1/3})$

- Therefore

$$\mathcal{J}_\mu^{\rm scalar} = \sqrt{\frac{1}{3}}\sqrt{\frac{1}{3}}\, e_u\, I_\mu^{\{ud\}} \neq 0$$

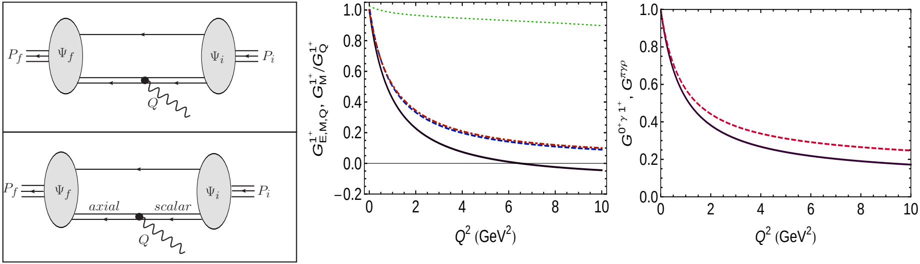
$$\mathcal{J}_\mu^{\rm axial} = \sqrt{\frac{2}{3}}\sqrt{\frac{2}{3}}\, e_d\, I_\mu^{\{uu\}} + \sqrt{\frac{1}{3}}\sqrt{\frac{1}{3}}\, e_u\, I_\mu^{\{ud\}} = 0$$



Hard contributions appear in the microscopic description of the elastic form factor of the proton

Remaining diagrams: Photon interacting with diquarks

H.L.L. Roberts *et al.* Phys. Rev. C **83**, 065206 (2011)



Composite object

↓

Electromagnetic radius is nonzero ($r_{qq} \gtrsim r_\pi$)

↓

Softer contribution to the form factors

Soft contributions appear in the microscopic description of the elastic form factor of the proton

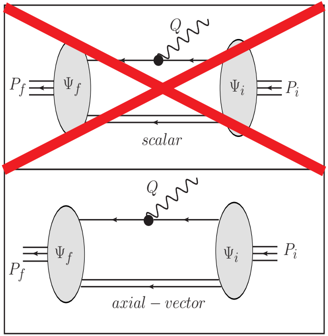
☞ Diagrams in which the photon interact with diquarks appear

Photon coupling directly to a dressed-quark with the diquark acting as a bystander

- Initial state: Proton
  - Two axial-vector diquark isospin states:

    $(I, I_z) = (1,1) \to$ flavor content: $\{uu\}$

    $(I, I_z) = (1,0) \to$ flavor content: $\{ud\}$
  - In the isospin limit, they appear with relative weighting: $(-\sqrt{2/3}) : (\sqrt{1/3})$
- Final state: Δ^+
  - Same isospin states of axial-vector diquark.
  - Different weighting due to $I_\Delta = 3/2$: $(\sqrt{1/3}) : (\sqrt{2/3})$



- Therefore

$$\mathcal{J}^{1,\text{axial}}_{\mu\alpha} = -\sqrt{\frac{2}{3}}\sqrt{\frac{1}{3}}\, e_d\, I^{1\{uu\}}_{\mu\alpha} + \sqrt{\frac{1}{3}}\sqrt{\frac{2}{3}}\, e_u\, I^{1\{ud\}}_{\mu\alpha} = \frac{\sqrt{2}}{3}\, I^{1\{qq\}}_{\mu\alpha}(K,Q)$$

Soft and still hard contributions appear in the microscopic description of the $\gamma^* N\Delta$ electromagnetic reaction

General observation

G_M^p vs G_M^{*p}

☞ *Similar contributions in both cases:*

G_M^{*p} *should fall asymptotically at the same rate as* G_M^p.

☞ *By isospin considerations:*

G_M^{*n} *should fall asymptotically at the same rate as* G_M^{*p}.

☞ *Hold SU(6):*

$$\langle p|\mu|\Delta^+\rangle \propto \langle n|\mu|\Delta^0\rangle \propto \langle p|\mu|p\rangle \,.$$

Symmetry preserving Dyson-Schwinger equation treatment of a vector ⊗ vector contact interaction

- Gluon propagator: Contact interaction.

$$g^2 D_{\mu\nu}(p-q) = \delta_{\mu\nu} \frac{4\pi\alpha_{\rm IR}}{m_{\rm G}^2}$$

- Truncation scheme: Rainbow-ladder.

$$\Gamma^a_\nu(q,p) = (\lambda^a/2)\gamma_\nu$$

- Fermion propagator: Gap equation.

$$S^{-1}(p) = i\gamma\cdot p + m + \Sigma(p)$$
$$= i\gamma\cdot p + M$$

  - $M \sim 0.4\,\rm GeV = constant$.
  - Implies momentum independent Bethe-Salpeter and Faddeev amplitudes.

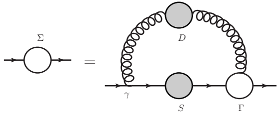
- Baryons: Faddeev equation.

$$m_N = 1.14\,\rm GeV \quad m_\Delta = 1.39\,\rm GeV$$

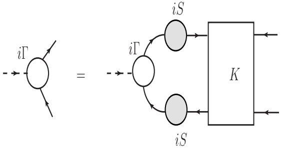
(masses reduced by pion-cloud effects)

- Ward-Green-Takahashi identities: Axial-vector and vector.

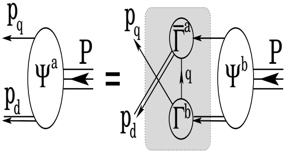
Gap equation



Bethe-Salpeter equation



Faddeev equation



Series of papers establishes strengths and limitations

Used judiciously, produces results indistinguishable from most sophisticated Rainbow-ladder interactions

- Spectrum of hadrons with strangeness
  Chen Chen, L. Chang, C.D. Roberts, Shaolong Wang and D.J. Wilson
  Few Body Syst. **53** 293-326 (2012). arXiv:1204.2553 [nucl-th]
- Nucleon and Roper electromagnetic elastic and transition form factors
  D.J. Wilson, I.C. Cloët, L. Chang and C.D. Roberts
  Phys. Rev. C **85**, 025205 (2012). arXiv:1112.2212 [nucl-th]
- π- and ρ-mesons, and their diquark partners, from a contact interaction
  H.L.L. Roberts, A. Bashir, L.X. Gutierrez-Guerrero, C.D. Roberts and D.J. Wilson
  Phys. Rev. C **83**, 065206 (2011). arXiv:1102.4376 [nucl-th]
- Masses of ground and excited-state hadrons
  H.L.L. Roberts, L. Chang, I.C. Cloët and C.D. Roberts
  Few Body Syst. **51**, 1-25 (2011). arXiv:1101.4244 [nucl-th]
- Abelian anomaly and neutral pion production
  H.L.L. Roberts, C.D. Roberts, A. Bashir, L.X. Gutierrez-Guerrero and P.C. Tandy
  Phys. Rev. C **82**, 065202 (2010). arXiv:1009.0067 [nucl-th]
- Pion form factor from a contact interaction
  L.X. Gutierrez-Guerrero, A. Bashir, I.C. Cloët and C.D. Roberts
  Phys. Rev. C **81**, 065202 (2010). arXiv:1002.1968 [nucl-th]

Weakness of contact-interaction

☞ Truncation which produces Faddeev amplitudes that are independent of relative momentum:

- Underestimates the quark orbital angular momentum content of the bound-state.
- Suppresses the two-loop diagrams.

	ind.-p DSE kernels	dep.-p DSE kernels
axial-diquark(Δ)-axial-diquark(p)	0.85	0.96
axial-diquark(Δ)-scalar-diquark(p)	0.18	1.27

Two sets of results

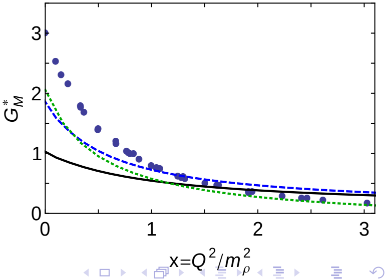
1. Original result.
2. Improved version:
   - Rescale the axial(Δ)-scalar(p) diagram

$$1 + \frac{g_{\rm as/aa}}{1 + Q^2/m_\rho^2}$$

$$\boxed{\text{axial}(\Delta)\text{-scalar}(p) = \text{axial}(\Delta)\text{-axial}(p)}$$

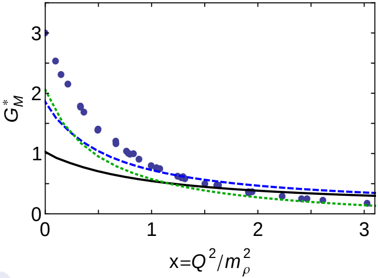
   - Incorporate dressed quark-anomalous magnetic moment
     - ☞ Consequence of the DCSB.

Coupled-channel prediction of the dressed quark core contribution



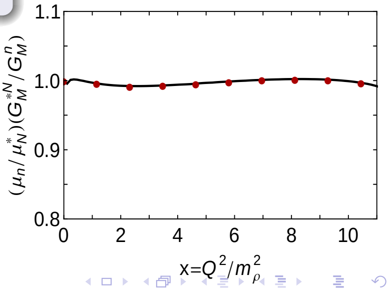
G^*_M vs. Experimental data and dynamical models

- Both computed curves are consistent with data for $Q^2 \gtrsim 2m_\rho^2$.
- They are in marked disagreement at infrared momenta.
- Similarity between our improved version and dressed-quark-core result determined by EBAC.
- The discrepancy results from the omission of meson-cloud effects.



Transition vs. elastic magnetic form factors

- The fall-off rate of $G^*_M(Q^2)$ in the $\gamma^* p \to \Delta^+$ transition must much that of $G_M(Q^2)$.
- With isospin symmetry, $\langle p|\mu|\Delta^+\rangle = -\langle n|\mu|\Delta^0\rangle$ is valid, so same is true of the $\gamma^* n \to \Delta^0$ magnetic form factor.
- These are statements about the dressed quark core contributions $\to$ Outside the domain of meson-cloud effects, $Q^2 \gtrsim 2\,\mathrm{GeV}^2$



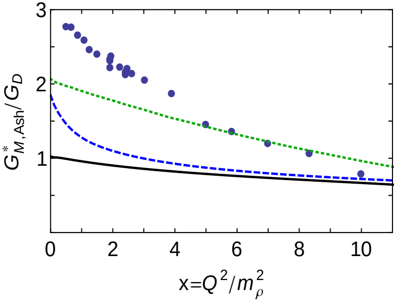
☞ Presentations of experimental data typically use the Ash convention.

- $G^*_{M,\mathrm{Ash}}(Q^2)$ falls faster than a dipole.
- Many have viewed this as a conundrum.
- There is no sound reason to expect:

$$G^*_{M,\mathrm{Ash}}/G^n_M \sim \mathrm{constant}$$

- Instead, the Jones-Scadron form factor should exhibit:

$$G^*_{M,\mathrm{J-S}}/G^n_M \sim \mathrm{constant}$$



Two main reasons

Meson-cloud effects

↓

Provide more than 30% for $Q^2 \lesssim 2m^2_\rho$

↓

These contributions are very soft

↓

They disappear rapidly

$G^*_{M,\mathrm{Ash}}$ vs. $G^*_{M,\mathrm{J-S}}$

$$G^*_{M,\mathrm{Ash}} = G^*_{M,\mathrm{J-S}} \left(1 + \frac{Q^2}{(m_\Delta + m_N)^2}\right)^{-\frac{1}{2}}$$

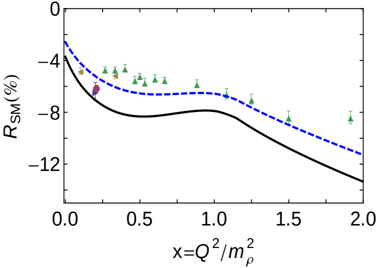
↓

A factor $1/Q$ of difference

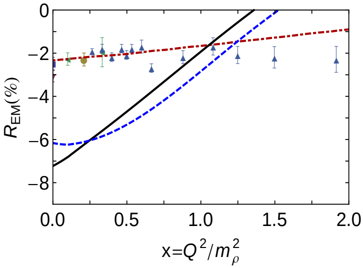
↓

Provides material damping for $Q^2 \gtrsim 4m^2_\rho$

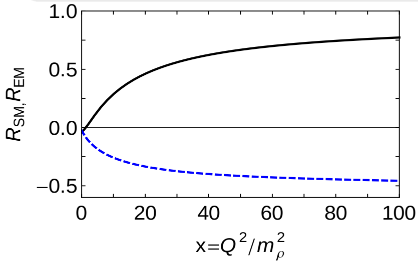
☞ R_{SM}: Good description of the rapid fall at large momentum transfer.



☞ R_{EM}: A particularly sensitive measure of orbital angular momentum correlations.



Large Q^2-behaviour of the quadrupole ratios



Observations:

- Truly asymptotic Q^2 is required before predictions are realized.
- $G_E^*(Q^2)$ posses a zero at an empirical accessible momentum and thereafter $R_{EM} \to 1$.
- $R_{SM} \to \mathrm{constant}$. The curve we display contains the $ln^2 Q^2$-growth expected in QCD.

Summary and Outlook

Epilogue:

We have computed the $\gamma^* N \to \Delta$ transition form factors using a Poincaré-covariant, symmetry-preserving treatment of a vector $\otimes$ vector contact interaction.

- Jones-Scadron G_M^{*p}:

  ☞ G_M^{*p} fall asymptotically at the same rate as G_M^p.

  ☞ Compatible with isospin symmetry and pQCD predictions.

  ☞ Data do not fall unexpectedly rapid once the kinematic relation between Jones-Scadron and Ash conventions is properly account for.

- R_{EM} and R_{SM}:

  ☞ Contact interaction produces correlations between dressed-quarks within Faddeev wave functions and related features in the current that are comparable in size with those observed empirically.

  ☞ Presence of a zero in the transition electric form factor.

  ☞ Limits of pQCD, $R_{EM} \to 1$ and $R_{SM} \to \text{constant}$, are apparent in our calculation but truly asymptotic Q^2 is required before the predictions are realized.

Outlook:

☞ *Verify claims using a more sophisticated interaction.*

☞ *Compute transition form factors for $N \to N(1535)S_{11}$.*

☞ *These projects are essential in paving the way for JLab to chart the infrared behaviour of QCD.*