THE NATURE OF THE ORBITALLY EXCITED
CHARMED-STRANGE MESONS THROUGH NONLEPTONIC
$B \rightarrow D^{(*)} D_{sJ}^{(*)}$ DECAYS

J. Segovia, C. Albertus, E. Hernández, F. Fernández and D.R. Entem
Grupo de Física Nuclear and IUFFyM
Universidad de Salamanca

12th International Workshop on Meson Production, Properties and Interaction
Kraków 31 May - 5 June 2012
## Table of Contents

1. Introduction

2. Constituent quark model

3. CQM predictions on $D_s P$-wave mesons

4. Nonleptonic $B$ decays into $D_s^*(J)D_s^{(*)}$ final states

5. Summary
1 Introduction

2 Constituent quark model

3 CQM predictions on $D_s$ $P$-wave mesons

4 Nonleptonic $B$ decays into $D^{(*)} D_s^{(*)}$ $sJ$ final states

5 Summary
**Experimental measurements**

**Nonleptonic decays of B mesons (at quark level $\bar{b} \rightarrow \bar{c}c\bar{s}$) have been used to search for new charmonium and charmed-strange mesons and to study their properties in detail.**

- The BaBar Collaboration found the $D_{s0}^*(2317)$ in the inclusive $D_s^+\pi^0$ invariant mass distribution.
- The CLEO Collaboration observed its doublet partner $D_{s1}(2460)$ in the $D_s^+\pi^0$ final state.

**The properties of these states were not well known until the Belle Collaboration observed the $B \rightarrow \bar{D}D_{s0}^*(2317)$ and $B \rightarrow \bar{D}D_{s1}(2460)$ decays.**

**Last measurements:** The Belle Collaboration in Phys. Rev. D83, 051102 (2011)

\[
R_{D^0} = \frac{\mathcal{B}(B \rightarrow DD_{s0}^*(2317))}{\mathcal{B}(B \rightarrow DD_s)} = 0.10 \pm 0.03, \quad R_{D^*0} = \frac{\mathcal{B}(B \rightarrow D^*D_{s0}^*(2317))}{\mathcal{B}(B \rightarrow D^*D_s)} = 0.15 \pm 0.06,
\]

\[
R_{D^1} = \frac{\mathcal{B}(B \rightarrow DD_{s1}(2460))}{\mathcal{B}(B \rightarrow DD_{s}^*)} = 0.44 \pm 0.11, \quad R_{D^*1} = \frac{\mathcal{B}(B \rightarrow D^*D_{s1}(2460))}{\mathcal{B}(B \rightarrow D^*D_{s}^*)} = 0.58 \pm 0.12,
\]

\[
R_{D^1'} = \frac{\mathcal{B}(B \rightarrow DD_{s1}(2536))}{\mathcal{B}(B \rightarrow DD_{s}^*)} = 0.049 \pm 0.010, \quad R_{D^*1'} = \frac{\mathcal{B}(B \rightarrow D^*D_{s1}(2536))}{\mathcal{B}(B \rightarrow D^*D_{s}^*)} = 0.044 \pm 0.010.
\]

**The study of nonleptonic decays can help to shed light on the structure of the $P$-wave charmed-strange mesons.**
Meson properties are characterized by the dynamics of the light quark:

\[ \vec{j}_q = \vec{L} + \vec{s}_q \]
\[ \vec{J} = \vec{j}_q + \vec{s}_Q \]

The lowest \( D_s P \)-wave mesons can be grouped into two doublets:

\[ j_q^P = \frac{1}{2} \rightarrow J^P = 0^+, 1^+ \]
\[ j_q^P = \frac{3}{2} \rightarrow J^P = 1^+, 2^+ \]

Properties:
- Doublets are degenerated.
- Strong decays of the \( D_{sJ}(j_q = 1/2) \) proceed only through \( S \)-waves ⇒ Broad states.
- Strong decays of the \( D_{sJ}(j_q = 3/2) \) proceed only through \( D \)-waves ⇒ Narrow states.
Spectroscopy of $D_s P$-wave mesons:

<table>
<thead>
<tr>
<th>$j_q^P = \frac{1}{2}^+$</th>
<th>$j_q^P = \frac{3}{2}^+$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$J^P = 0^+$</td>
<td>$D_{s0}^*(2317)$</td>
</tr>
<tr>
<td></td>
<td>$2317.4 \pm 0.9$</td>
</tr>
<tr>
<td>$J^P = 1^+$</td>
<td>$D_{s1}(2536)$</td>
</tr>
<tr>
<td></td>
<td>$2535.3 \pm 0.6$</td>
</tr>
<tr>
<td>$J^P = 2^+$</td>
<td>$D_{s2}^*(2573)$</td>
</tr>
<tr>
<td></td>
<td>$2572.4 \pm 1.5$</td>
</tr>
</tbody>
</table>

Nonleptonic decay ratios of $D_s P$-wave mesons:

- **Factorization approximation:**
  - Matrix element of $B \rightarrow D^{(*)} D_{sJ}^{(*)}$ as a product of two matrix elements:
    1. The $B$ weak transition into the $D^{(*)}$ mesons.
    2. The weak creation of the $c\bar{s}$ pair which makes the $D_{sJ}^{(*)}$ meson $\propto f_{D_{sJ}^{(*)}}$.

- **Heavy quark limit:**
  1. PS effects are neglected: $R_{D0} = R_{D^*0} = \left| \frac{f_{D_{s0}^*(2317)}}{f_{D_{s0}}} \right|^2$, $R_{D1} = R_{D^*1} = \left| \frac{f_{D_{s1}(2460)}}{f_{D_{s1}}} \right|^2$.
  2. $f_{D_{s0}^*} = f_{D_{s1}}$ and $f_{D_s} = f_{D_{s}^*} \Rightarrow R_{D0} \approx R_{D1}$.
  3. For $P$-wave $j_q = 1/2$ states $\rightarrow$ decay constants very similar to those of $D_s$ and $D_s^*$.
  4. For $P$-wave $j_q = 3/2$ states $\rightarrow$ decay constants very small.

**Ratios of order one** for $D_{s0}^*(2317)$ and $D_{s1}(2460)$ $\rightarrow$ in strong disagreement

**We will concentrate in the influence of the effect of the finite $c$-quark mass**
Table of Contents

1 Introduction

2 Constituent quark model

3 CQM predictions on $D_s$ $P$-wave mesons

4 Nonleptonic $B$ decays into $D_s^{(*)} D_{sJ}^{(*)}$ final states

5 Summary
Goldstone-boson exchange potentials

- QCD Lagrangian invariant under the chiral transformation

\[ \mathcal{L} = \bar{\psi} \left( i \gamma^\mu \partial_\mu - M(q^2) U^{\gamma_5} \right) \psi \]

Chiral symmetry is spontaneously broken

- Pseudo-Goldstone Bosons (\( \vec{\pi}, K_i \) and \( \eta_8 \))

\[ U^{\gamma_5} = \exp \left( i \frac{\pi^a \lambda^a \gamma_5}{f_\pi} \right) \]

\[ \sim 1 + \frac{i}{f_\pi} \gamma^5 \lambda^a \pi^a - \frac{1}{2f_\pi^2} \pi^a \pi^a + \ldots \]

- Constituent quark mass

\[ M(q^2) = m_q F(q^2) = m_q \left[ \frac{\Lambda^2}{\Lambda^2 + q^2} \right]^{1/2} \]

We take it into account through the one-gluon exchange (OGE) potential.

The OGE is a standard color Fermi-Breit interaction obtained from the vertex Lagrangian:

\[ \mathcal{L}_{qqg} = i \sqrt{4\pi \alpha_s} \bar{\psi} \gamma_{\mu} G_{\mu}^{\mu} \lambda^c \psi \]

Effective scale dependent strong coupling constant:

\[ \alpha_s(\mu) = \frac{\alpha_0}{\ln \left( \frac{\mu^2 + \mu_0^2}{\Lambda_0^2} \right)} \]

**Confinement potential**

![Graph showing the confinement potential with quenched κ = 0.1575, \( \Sigma^+ \) and \( \Pi_u \) states, and linear screened potential equation.]

\[ V_{\text{CON}}(r) = \left[ -a_c(1 - e^{-\mu_c r}) + \Delta \right] (\vec{\lambda}_i \cdot \vec{\lambda}_j) \]

- **Flavor independent**
- \( r \to 0 \Rightarrow V_{\text{CON}}(r) \to (-a_c \mu_c r + \Delta)(\vec{\lambda}_i \cdot \vec{\lambda}_j) \)
- \( r \to \infty \Rightarrow V_{\text{CON}}(r) \to (-a_c + \Delta)(\vec{\lambda}_i \cdot \vec{\lambda}_j) \)

---

**Introduction**

- Constituent quark model (CQM)
- Predictions on \( D_s \) \( P \)-wave mesons
- Nonleptonic \( B \) decays into \( D(\ast)D(\ast)_{sJ} \) final states

**Summary**

**LINEAR SCREENED POTENTIAL**


Some recent applications

Deuteron

<table>
<thead>
<tr>
<th></th>
<th>CQM</th>
<th>NijmII</th>
<th>Bonn B</th>
<th>Exp.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\epsilon_d$ (MeV)</td>
<td>-2.2242</td>
<td>-2.2246</td>
<td>-2.2246</td>
<td>-2.224575</td>
</tr>
<tr>
<td>$P_d$ (%)</td>
<td>4.85</td>
<td>5.64</td>
<td>4.99</td>
<td>-</td>
</tr>
<tr>
<td>$Q_d$ (fm$^2$)</td>
<td>0.276</td>
<td>0.271</td>
<td>0.278</td>
<td>0.285 ± 0.0003</td>
</tr>
<tr>
<td>$A_S$ (fm$^{-1/2}$)</td>
<td>0.891</td>
<td>0.8845</td>
<td>0.8860</td>
<td>0.8846 ± 0.0009</td>
</tr>
<tr>
<td>$A_D/A_S$</td>
<td>0.0257</td>
<td>0.0252</td>
<td>0.0264</td>
<td>0.0256 ± 0.0004</td>
</tr>
</tbody>
</table>

Light mesons

Reactions

X(3872)

The nature of the orbitally excited charmed-strange mesons

Jorge Segovia et al., segonza@usal.es
Table of Contents

1 Introduction

2 Constituent quark model

3 CQM predictions on $D_s$ $P$-wave mesons

4 Nonleptonic $B$ decays into $D_s^{(*)} D_s^{(*)} J$ final states

5 Summary
What is the mechanism that explains the lower mass of the $D_{s0}^*(2317)$ meson?

Addition of the one-loop QCD corrections to the spin-dependent terms of the potential.

There is a spin-dependent term which affects only to mesons with different flavor quarks.


Our prediction:

<table>
<thead>
<tr>
<th></th>
<th>$j_q^P = 1/2^-$</th>
<th>$j_q^P = 1/2^+$</th>
<th>$j_q^P = 3/2^+$</th>
</tr>
</thead>
<tbody>
<tr>
<td>This work ($\alpha_s$)</td>
<td>1984 2110 2510 2593 2554 2591</td>
<td>1984 2104 2383 2570 2560 2609</td>
<td>0$^-$ 1$^-$ 0$^+$ 1$^+$ 1$^+$ 2$^+$</td>
</tr>
<tr>
<td>This work ($\alpha_s^2$)</td>
<td>1969.0 ± 1.4 2112.3 ± 0.5 2318.0 ± 1.0 2459.6 ± 0.9 2535.12 ± 0.25 2572.6 ± 0.9</td>
<td>1896 2017 2516 2596 2466 2513</td>
<td>1896 2014 2362 2535 2499 2544</td>
</tr>
<tr>
<td>Exp.</td>
<td>1867.7 ± 0.3 2010.25 ± 0.14 2403 ± 38 2427 ± 36 2423.4 ± 3.1 2460.1 ± 4.4</td>
<td>1867.7 ± 0.3 2010.25 ± 0.14 2403 ± 38 2427 ± 36 2423.4 ± 3.1 2460.1 ± 4.4</td>
<td>0$^+$ states are more sensitive to the inclusion of the one-loop corrections.</td>
</tr>
</tbody>
</table>
**Introduction**

Constituent quark model

CQM predictions on $D_{s1}$ $P$-wave mesons

Nonleptonic $B$ decays into $D(\ast)/D_{sJ}(\ast)$ final states

**Summary**

---

**$D_{s1}(2460)$ and $D_{s1}(2536)$. The tetraquark coupling**

*What is the mechanism that explains the lower mass of the $D_{s1}(2460)$ meson?*

- Possible molecular states.
- Tetraquark states.
- $c\bar{s}$ states in models with higher order corrections.
- Mixture of $c\bar{s}$ and $c\bar{s}n\bar{n}$.

**COUPLING PRESCRIPTION**  

- Working in HQS limit:
  - Three different spin states for tetraquark: $|0\ 1/2\rangle$, $|1\ 1/2\rangle$ and $|1\ 3/2\rangle$. The first index denotes the spin of the $n\bar{n}$ pair and the second one the coupling with the $\bar{s}$ spin.

- $^3P_0$ model to select the dominant couplings:
  - The $n\bar{n}$ pair created is in a $J = 0$ state.
  - The coupling between $D_s$ states and $|0\ 1/2\rangle$ tetraquark component is dominant.

- This choice has several advantages:
  - It has the correct heavy quark limit.
  - It can reproduce the narrow width of the $D_{s1}(2536)$ meson.
  - It is in agreement with the experimental situation which tells us that the prediction of the heavy quark limit is reasonable for the $j_q = 3/2$ states but not for the $j_q = 1/2$.  

---

Jorge Segovia et al., segonza@usal.es  
The nature of the orbitally excited charmed-strange mesons
**Introduction**

**Constituent quark model**

CQM predictions on $D_s$ \( P \)-wave mesons

Nonleptonic $B$ decays into $D(\ast)\bar{D}(\ast)$ final states

**Summary**

$D_{s1}(2460)$ and $D_{s1}(2536)$. The tetraquark coupling

Effective coupling between $c\bar{s}$ states and the tetraquark:

$$M = \begin{pmatrix}
M_{3P_1} & C_{SO} & \sqrt{\frac{2}{3}} C_S \\
C_{SO} & M_{1P_1} & \sqrt{\frac{1}{3}} C_S \\
\sqrt{\frac{2}{3}} C_S & \sqrt{\frac{1}{3}} C_S & M_{c\bar{s}n\bar{n}}
\end{pmatrix}$$

Eigenstates are:

<table>
<thead>
<tr>
<th>$M$ (MeV)</th>
<th>$S(^3P_1)$</th>
<th>$P(^3P_1)$</th>
<th>$S(^1P_1)$</th>
<th>$P(^1P_1)$</th>
<th>$S(c\bar{s}n\bar{n})$</th>
<th>$P(c\bar{s}n\bar{n})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2459</td>
<td>-</td>
<td>55.7</td>
<td>-</td>
<td>18.8</td>
<td>+</td>
<td>25.5</td>
</tr>
<tr>
<td>2557</td>
<td>+</td>
<td>27.7</td>
<td>-</td>
<td>72.1</td>
<td>+</td>
<td>0.2</td>
</tr>
<tr>
<td>2973</td>
<td>+</td>
<td>16.6</td>
<td>+</td>
<td>9.1</td>
<td>+</td>
<td>74.3</td>
</tr>
</tbody>
</table>

The doublets $D_{sJ}$ with $j_q = 1/2, 3/2$ from HQS:

$$|1/2, 0^+\rangle = |^3P_0\rangle$$

$$|3/2, 1^+\rangle = \sqrt{\frac{1}{3}} |^3P_1\rangle - \sqrt{\frac{2}{3}} |^1P_1\rangle$$

$$|1/2, 1^+\rangle = \sqrt{\frac{2}{3}} |^3P_1\rangle + \sqrt{\frac{1}{3}} |^1P_1\rangle$$

$$|3/2, 2^+\rangle = |^3P_2\rangle$$
**$D_{s1}(2460)$ and $D_{s1}(2536)$. The tetraquark coupling**

- Calculation of the following physical observables:

  \[
  \Gamma(D_{s1}(2536)^+) = \Gamma(D^{*0}K^+) + \Gamma(D^{*+}K^0)
  \]

  \[
  R_1 = \frac{\Gamma(D_{s1}(2536)^+ \rightarrow D^{*0}K^+)}{\Gamma(D_{s1}(2536)^+ \rightarrow D^{*+}K^0)}
  \]

  \[
  R_2 = \frac{\Gamma_S(D_{s1}(2536)^+ \rightarrow D^{*+}K^0)}{\Gamma(D_{s1}(2536)^+ \rightarrow D^{*+}K^0)}
  \]

  \[
  R_3 = \frac{\Gamma(D_{s1}(2536)^+ \rightarrow D^{+}\pi^-K^+)}{\Gamma(D_{s1}(2536)^+ \rightarrow D^{*+}K^0)}
  \]

- $D_{s1}(2536)$ Eigenstate:

<table>
<thead>
<tr>
<th>Mass (MeV)</th>
<th>$\Gamma$ (MeV)</th>
<th>$R_1$</th>
<th>$R_2$</th>
<th>$R_3$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2557</td>
<td>0.46</td>
<td>1.31</td>
<td>0.66</td>
<td>4.00</td>
</tr>
<tr>
<td>Exp.</td>
<td>1.03</td>
<td>1.27</td>
<td>0.72</td>
<td>3.27</td>
</tr>
</tbody>
</table>

As the $DK$ decay is zero the total width would be mainly given by the $D^*K$ channel and is in the order of the experimental value.
Semileptonic $B$ ($B_s$) decays into $D^{**}$ ($D_s^{**}$)

*Different collaborations have recently reported semileptonic $B$ decays into orbitally excited charmed mesons.*

*The theoretical analysis of these data offers the possibility for a stringent test of meson models.*

*Include a weak decay of the $B$ meson and a strong decay of the $D$ meson.*

**Weak decays:**
- Matrix elements $\Rightarrow$ parametrized in terms of form factors.
- At the quark level $\Rightarrow$ light quark is a spectator.
- We follow references below:

**Strong decays:**
- The $^3P_0$ decay model.
- The microscopic decay model.
Semileptonic $B_s$ decays into $D_{s}^{**}$

<table>
<thead>
<tr>
<th>$D_{s0}^{*}(2317)$</th>
<th>Experiment $(\times 10^{-3})$</th>
<th>Theory $(\times 10^{-3})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mathcal{B}(B_s^0 \to D_{s0}^{*}(2317)^- \mu^+ \nu_{\mu})$</td>
<td>-</td>
<td>4.4282</td>
</tr>
<tr>
<td>$D_{s1}(2460)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\mathcal{B}(B_s^0 \to D_{s1}(2460)^- \mu^+ \nu_{\mu})$</td>
<td>-</td>
<td>1.74 – 5.70</td>
</tr>
<tr>
<td>$D_{s1}(2536)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\mathcal{B}(B_{s}^{0} \to D_{s1}(2536)^{-} \mu^{+} \nu_{\mu})\mathcal{B}(D_{s1}(2536)^{-} \to D^{*-}\bar{K}^{0})$</td>
<td>2.4 ± 0.7</td>
<td>2.0491 2.2397</td>
</tr>
<tr>
<td>$D_{s2}^{*}(2573)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\mathcal{B}(B_{s}^{0} \to D_{s2}^{<em>}(2573)^{-} \mu^{+} \nu_{\mu})\mathcal{B}(D_{s2}^{</em>}(2573)^{-} \to D^{-}\bar{K}^{0})$</td>
<td>-</td>
<td>1.7047 1.7680</td>
</tr>
<tr>
<td>$\mathcal{B}(B_{s}^{0} \to D_{s2}^{<em>}(2573)^{-} \mu^{+} \nu_{\mu})\mathcal{B}(D_{s2}^{</em>}(2573)^{-} \to D^{*-}\bar{K}^{0})$</td>
<td>-</td>
<td>0.1769 0.1136</td>
</tr>
<tr>
<td>$\mathcal{B}(B_{s}^{0} \to D_{s2}^{<em>}(2573)^{-} \mu^{+} \nu_{\mu})\mathcal{B}(D_{s2}^{</em>}(2573)^{-} \to D^{(*)-}\bar{K}^{0})$</td>
<td>-</td>
<td>1.8816 1.8816</td>
</tr>
</tbody>
</table>

Jorge Segovia et al., segonza@usal.es

The nature of the orbitally excited charmed-strange mesons
Table of Contents

1 Introduction

2 Constituent quark model

3 CQM predictions on $D_s P$-wave mesons

4 Nonleptonic $B$ decays into $D^{(*)}D^{(*)}_{sJ}$ final states

5 Summary
These decays provide valuable information that makes possible to check the structure of the $D_{s0}^*(2317)$, $D_{s1}(2460)$ and $D_{s1}(2536)$ mesons

Heavy Quark Symmetry predictions

- Ratios of order one for $D_{s0}^*(2317)$ and $D_{s1}(2460)$ ⇒ in strong disagreement.
- Ratios very small for $D_{s1}(2536)$ ⇒ follow the expectations.

The disagreement ⇒ an indication that $D_{s0}^*(2317)$ and $D_{s1}(2460)$ mesons could have a more complex structure

Expressions of the decay widths taking finite $c$-quark mass

For $D_{sJ}^{(*)}$ a pseudoscalar or scalar meson:

$$
\Gamma = \frac{G_F^2}{16\pi m_B^2} |V_{cb}|^2 |V_{cs}|^2 a_1^2 \frac{\lambda^{1/2}(m_B^2, m_{D^*}^2, m_{D^{(*)}}^2)}{2m_B} m_{D_{sJ}^{(*)}}^2 f_{D_{sJ}^{(*)}}^2 \mathcal{H}_{tt}^{B\rightarrow D^{(*)}}(m_{D_{sJ}^{(*)}}^2)
$$

For $D_{sJ}^{(*)}$ a vector or axial-vector meson:

$$
\Gamma = \frac{G_F^2}{16\pi m_B^2} |V_{cb}|^2 |V_{cs}|^2 a_1^2 \frac{\lambda^{1/2}(m_B^2, m_{D^*}^2, m_{D^{(*)}}^2)}{2m_B} m_{D_{sJ}^{(*)}}^2 f_{D_{sJ}^{(*)}}^2 \times
$$

$$
\times \left[ \mathcal{H}_{+1+1}^{B\rightarrow D^{(*)}}(m_{D_{sJ}^{(*)}}^2) + \mathcal{H}_{-1-1}^{B\rightarrow D^{(*)}}(m_{D_{sJ}^{(*)}}^2) + \mathcal{H}_{00}^{B\rightarrow D^{(*)}}(m_{D_{sJ}^{(*)}}^2) \right]
$$
Discussion on the phase space

- Using experimental masses we obtain the ratios:

\[
R_{D_0} = 0.9008 \times \left| \frac{f_{D^*_s(2317)}}{f_{D_s}} \right|^2, \quad R_{D_1} = 0.7040 \times \left| \frac{f_{D_{s1}(2460)}}{f_{D^*_s}} \right|^2, \quad R_{D_1'} = 0.6370 \times \left| \frac{f_{D_{s1}(2536)}}{f_{D^*_s}} \right|^2
\]

\[
R_{D^*_0} = 0.7166 \times \left| \frac{f_{D^*_s(2317)}}{f_{D_s}} \right|^2, \quad R_{D^*_1} = 1.0039 \times \left| \frac{f_{D_{s1}(2460)}}{f_{D^*_s}} \right|^2, \quad R_{D^*_1'} = 0.9923 \times \left| \frac{f_{D_{s1}(2536)}}{f_{D^*_s}} \right|^2
\]

The double ratio does not depend on decay constants:

\[
\frac{R_{D^*_0}}{R_{D_0}} = \begin{cases} 
0.7955 \\ 1.50 \pm 0.75 
\end{cases}, \quad \frac{R_{D^*_1}}{R_{D_1}} = \begin{cases} 
1.4260 \\ 1.32 \pm 0.43 
\end{cases}, \quad \frac{R_{D^*_1'}}{R_{D_1'}} = \begin{cases} 
1.5578 \\ 0.90 \pm 0.27 
\end{cases}
\]

- The quality of the experimental results does not allow to be very conclusive as to the goodness of the factorization approximation.

- But one can conclude that phase space and weak matrix element corrections cannot be ignored.
Discussion on the decay constants

We will use for the pseudoscalar constant the experimental value.

<table>
<thead>
<tr>
<th>Decay constants of $^{3}S_{1}$ mesons agree.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Decay constants of $^{1}S_{0}$ mesons are large:</td>
</tr>
<tr>
<td>i) A spin-spin contact hyperfine interaction regularized at $r = 0$.</td>
</tr>
<tr>
<td>ii) Hyperfine splittings in the different flavor sectors are well reproduced.</td>
</tr>
<tr>
<td>iii) Most of the physical observables are insensitive to the regularization → BUT → those related with annihilation processes are affected.</td>
</tr>
<tr>
<td>iv) The effect is:</td>
</tr>
<tr>
<td>• Negligible for higher partial waves.</td>
</tr>
<tr>
<td>• Small in the $^{3}S_{1}$ channel.</td>
</tr>
<tr>
<td>• Sizable in the $^{1}S_{0}$ channel.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Approach</th>
<th>$f_{D}$ (MeV)</th>
<th>$f_{D_{s}}$ (MeV)</th>
<th>$f_{D_{s}}/f_{D}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ours</td>
<td>297.019$^{(†)}$</td>
<td>416.827$^{(†)}$</td>
<td>1.40$^{(†)}$</td>
</tr>
<tr>
<td></td>
<td>214.613$^{(‡)}$</td>
<td>286.382$^{(‡)}$</td>
<td>1.33$^{(‡)}$</td>
</tr>
<tr>
<td>Experiment</td>
<td>206.7 ± 8.9</td>
<td>257.5 ± 6.1</td>
<td>1.25 ± 0.06</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Approach</th>
<th>$f_{D^{*}}$ (MeV)</th>
<th>$f_{D_{s}^{*}}$ (MeV)</th>
<th>$f_{D_{s}^{<em>}}/f_{D^{</em>}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ours</td>
<td>247.865$^{(†)}$</td>
<td>329.441$^{(†)}$</td>
<td>1.33$^{(†)}$</td>
</tr>
<tr>
<td>QL (Italy)</td>
<td>234</td>
<td>254</td>
<td>1.04 ± 0.01$^{+2}_{-4}$</td>
</tr>
<tr>
<td>QL (UKQCD)</td>
<td>245 ± 20$^{+0}_{-2}$</td>
<td>272 ± 16$^{+0}_{-20}$</td>
<td>1.11 ± 0.03</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Decay constants of $D_{s0}^{*}(2317)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_{D_{s0}^{*}(2317)} \sim 1/3 f_{D_{s}}$</td>
</tr>
<tr>
<td>$f_{D_{s0}^{*}(2317)} \sim f_{D_{s1}(2460)}$</td>
</tr>
<tr>
<td>$f_{D_{s1}(2536)} \rightarrow$ small</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>From experiment:</td>
</tr>
<tr>
<td>$f_{D_{s0}^{*}(2317)} = 0.72 f_{D_{s1}(2460)}$</td>
</tr>
<tr>
<td>$f_{D_{s1}(2536)} = 59.176$ MeV</td>
</tr>
</tbody>
</table>

Jorge Segovia et al., segona@usal.es

The nature of the orbitally excited charmed-strange mesons
Our results

<table>
<thead>
<tr>
<th>Ratios</th>
<th>$X \equiv D_{s0}^*(2317)$</th>
<th>$X \equiv D_{s1}(2460)$</th>
<th>$X \equiv D_{s1}(2536)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mathcal{B}(B \to DX)/\mathcal{B}(B \to DD_s)$</td>
<td>0.19(*)</td>
<td>0.10 ± 0.03</td>
<td>-</td>
</tr>
<tr>
<td>$\mathcal{B}(B \to D^*X)/\mathcal{B}(B \to D^*D_s)$</td>
<td>0.15(*)</td>
<td>0.15 ± 0.06</td>
<td>-</td>
</tr>
<tr>
<td>$\mathcal{B}(B \to DX)/\mathcal{B}(B \to DD_s^*)$</td>
<td>-</td>
<td>-</td>
<td>$[0.176^{(1)}_{0.177^{(2)}}]$</td>
</tr>
<tr>
<td>$\mathcal{B}(B \to D^*X)/\mathcal{B}(B \to D^<em>D_s^</em>)$</td>
<td>-</td>
<td>-</td>
<td>$[0.251^{(1)}_{0.252^{(2)}}]$</td>
</tr>
</tbody>
</table>

Comments

- Results close to or within the experimental error bars for the $D_{s0}^*(2317)$ meson.
  - 1-loop corrections to OGE has been introduced.
  - (*) We are using the experimental value for $f_{D_s}$.

- The enhancement of the $j_q = 3/2$ component of the $D_{s1}(2536)$ meson gives rise to ratios in better agreement with experiment.
  - (1) Without coupling to non-$q\bar{q}$ degrees of freedom $\to$ second ratio in disagreement.
  - (2) With coupling to non-$q\bar{q}$ degrees of freedom $\to$ $D_{s1}(2536)$ remains almost a $q\bar{q}$.

- The $D_{s1}(2460)$ meson could have a sizable non-$q\bar{q}$ component.
  - (1) Without coupling to non-$q\bar{q}$ degrees of freedom $\to$ ratios in disagreement.
  - (2) With coupling to non-$q\bar{q}$ degrees of freedom $\to$ ratios in disagreement $\to$ We have not yet calculated the contribution of the non-$q\bar{q}$ degrees of freedom.
<table>
<thead>
<tr>
<th>Chapter</th>
<th>Title</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Introduction</td>
</tr>
<tr>
<td>2</td>
<td>Constituent quark model</td>
</tr>
<tr>
<td>3</td>
<td>CQM predictions on $D_s$ $P$-wave mesons</td>
</tr>
<tr>
<td>4</td>
<td>Nonleptonic $B$ decays into $D^{(<em>)}D_{sJ}^{(</em>)}$ final states</td>
</tr>
<tr>
<td>5</td>
<td>Summary</td>
</tr>
</tbody>
</table>
Ratios of nonleptonic B decays into double charmed mesons have been recently reported by the Belle Collaboration.

The strong disagreement found between the heavy quark limit predictions and the experimental data motivates the introduction of the finite c-quark mass effects.

We have performed a calculation of the ratios working within the framework of the constituent quark model and in the factorization approximation.

The $D_{s0}(2317)$ meson:
- The mass is lowered towards the experimental value with the inclusion of the 1-loop corrections to the OGE potential.
- We obtain ratios compatible with the experimental data.
- Our results indicate that this meson could be described as a canonical $c\bar{s}$ state.

The $D_{s1}(2536)$ meson:
- We incorporate the non-$q\bar{q}$ degrees of freedom in the $J^P = 1^+$ channel.
- This meson remains almost a pure $q\bar{q}$ state and its $j_q = 3/2$ component is enhanced.
- Correct ratios for this meson are predicted.

The $D_{s1}(2460)$ meson:
- Has a sizable non-$q\bar{q}$ component.
- The non-$q\bar{q}$ contribution has not been calculated.
- The ratios are a factor 2 below the experimental ones.