Charmonium properties in a renormalization scheme

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Meson-Nucleon Physics and the Structure of the Nucleon

University of Granada

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Contents

1.- Constituent quark model

1.1.- Most important ingredients / J. Phys. G: Nucl. Part. Phys. **31**, 481 (2005)

- Spontaneous chiral symmetry breaking (Goldstone-Bosons exchange):

$$L = \bar{\psi}\left(i\gamma^\mu \partial_\mu - M U^{\gamma_5}\right)\psi \rightarrow \quad U^{\gamma_5} = 1 + \frac{i}{f_\pi}\gamma^5 \lambda^a \pi^a - \frac{1}{2f_\pi^2}\pi^a\pi^a + \ldots$$

$$M(q^2) = m_q F\left(q^2\right) = m_q \left[\frac{\Lambda^2}{\Lambda^2 + q^2}\right]^{1/2}$$

- QCD perturbative effects (One gluon exchange):

$$L = i\sqrt{4\pi\alpha_s}\,\bar{\psi}\gamma_\mu G^\mu \lambda^c \psi$$

- Confinement (screened potential):

$$V_{CON}^{C}(\vec{r}_{ij}) = \left[-a_c(1 - e^{-\mu_c r_{ij}}) + \Delta\right](\vec{\lambda}_i^c \cdot \vec{\lambda}_j^c)$$

$$\begin{cases} V_{CON}^{C}(\vec{r}_{ij}) = (-a_c \mu_c r_{ij} + \Delta)(\vec{\lambda}_i^c \cdot \vec{\lambda}_j^c) & r_{ij} \longrightarrow 0 \\ V_{CON}^{C}(\vec{r}_{ij}) = (-a_c + \Delta)(\vec{\lambda}_i^c \cdot \vec{\lambda}_j^c) & r_{ij} \longrightarrow \infty \end{cases}$$

Remember: $\sigma = -a_c \mu_c (\vec{\lambda}_i^c \cdot \vec{\lambda}_j^c)$

1.2.- Non-relativistic reduction of our potential

- One-gluon exchange (OGE)

$$V_{OGE}^{C}(\vec{r}_{ij}) = \frac{1}{4}\alpha_s(\vec{\lambda}_i^c \cdot \vec{\lambda}_j^c)\left[\frac{1}{r_{ij}} - \frac{1}{6m_i m_j}(\vec{\sigma}_i \cdot \vec{\sigma}_j)\frac{e^{-r_{ij}/r_0(\mu)}}{r_{ij} r_0^2(\mu)}\right]$$

$$V_{OGE}^{T}(\vec{r}_{ij}) = -\frac{1}{16}\frac{\alpha_s}{m_i m_j}(\vec{\lambda}_i^c \cdot \vec{\lambda}_j^c)\left[\frac{1}{r_{ij}^3} - \frac{e^{-r_{ij}/r_g(\mu)}}{r_{ij}}\left(\frac{1}{r_{ij}^2} + \frac{1}{3r_g^2(\mu)} + \frac{1}{r_{ij} r_g(\mu)}\right)\right] S_{ij}$$

$$V_{OGE}^{SO}(\vec{r}_{ij}) = -\frac{1}{16}\frac{\alpha_s}{m_i^2 m_j^2}(\vec{\lambda}_i^c \cdot \vec{\lambda}_j^c)\left[\frac{1}{r_{ij}^3} - \frac{e^{-r_{ij}/r_g(\mu)}}{r_{ij}^3}\left(1 + \frac{r_{ij}}{r_g(\mu)}\right)\right] \times$$
$$\times \left[((m_i + m_j)^2 + 2m_i m_j)(\vec{S}_+ \cdot \vec{L}) + (m_j^2 - m_i^2)(\vec{S}_- \cdot \vec{L})\right]$$

- Confinement

$$V_{CON}^{C}(\vec{r}_{ij}) = \left[-a_c(1 - e^{-\mu_c r_{ij}}) + \Delta\right](\vec{\lambda}_i^c \cdot \vec{\lambda}_j^c)$$

$$V_{CON}^{SO}(\vec{r}_{ij}) = -\left(\vec{\lambda}_i^c \cdot \vec{\lambda}_j^c\right)\frac{a_c \mu_c e^{-\mu_c r_{ij}}}{4m_i^2 m_j^2 r_{ij}}\left[((m_i^2 + m_j^2)(1 - 2a_s)\right.$$
$$\left. + 4m_i m_j(1 - a_s))(\vec{S}_+ \cdot \vec{L}) + (m_j^2 - m_i^2)(1 - 2a_s)(\vec{S}_- \cdot \vec{L})\right]$$

1.2.- Non-relativistic reduction of our potential. Singular contributions

$$V_{OGE}^{C}(\vec{r}_{ij}) = \frac{1}{4}\alpha_s(\vec{\lambda}_i^c\cdot\vec{\lambda}_j^c)\left[\frac{1}{r_{ij}} - \frac{1}{6m_im_j}(\vec{\sigma}_i\cdot\vec{\sigma}_j)4\pi\delta(r_{ij})\right]$$

$$V_{OGE}^{T}(\vec{r}_{ij}) = -\frac{1}{16}\frac{\alpha_s}{m_im_j}(\vec{\lambda}_i^c\cdot\vec{\lambda}_j^c)\frac{1}{r_{ij}^3}S_{ij}$$

$$V_{OGE}^{SO}(\vec{r}_{ij}) = -\frac{1}{16}\frac{\alpha_s}{m_i^2m_j^2}(\vec{\lambda}_i^c\cdot\vec{\lambda}_j^c)\frac{1}{r_{ij}^3}\times \times\left[((m_i+m_j)^2+2m_im_j)(\vec{S}_+\cdot\vec{L})+(m_j^2-m_i^2)(\vec{S}_-\cdot\vec{L})\right]$$

$$V_{OGE}^{C}(\vec{r}_{ij}) = \frac{1}{4}\alpha_s(\vec{\lambda}_i^c\cdot\vec{\lambda}_j^c)\left[\frac{1}{r_{ij}} - \frac{1}{6m_im_j}(\vec{\sigma}_i\cdot\vec{\sigma}_j)\frac{e^{-r_{ij}/r_0(\mu)}}{r_{ij}r_0^2(\mu)}\right]$$

$$V_{OGE}^{T}(\vec{r}_{ij}) = -\frac{1}{16}\frac{\alpha_s}{m_im_j}(\vec{\lambda}_i^c\cdot\vec{\lambda}_j^c)\left[\frac{1}{r_{ij}^3} - \frac{e^{-r_{ij}/r_g(\mu)}}{r_{ij}}\left(\frac{1}{r_{ij}^2}+\frac{1}{3r_g^2(\mu)}+\frac{1}{r_{ij}r_g(\mu)}\right)\right]S_{ij}$$

$$V_{OGE}^{SO}(\vec{r}_{ij}) = -\frac{1}{16}\frac{\alpha_s}{m_i^2m_j^2}(\vec{\lambda}_i^c\cdot\vec{\lambda}_j^c)\left[\frac{1}{r_{ij}^3} - \frac{e^{-r_{ij}/r_g(\mu)}}{r_{ij}^3}\left(1+\frac{r_{ij}}{r_g(\mu)}\right)\right]\times \times\left[((m_i+m_j)^2+2m_im_j)(\vec{S}_+\cdot\vec{L})+(m_j^2-m_i^2)(\vec{S}_-\cdot\vec{L})\right]$$

1.3.- Some recent applications

- **N-N interaction**
  - D. R. Entem, F. Fernández and A. Valcarce, Phys. Rev. C **62**, 034002 (2000)
  - B. Julia-Diaz, J. Haidenbauer, A. Valcarce and F. Fernández, Phys. Rev. C **65**, 034001 (2002)
- **Baryon spectrum**
  - H. Garcilazo, A. Valcarce and F. Fernández, Phys. Rev. C **63**, 035207 (2001)
  - H. Garcilazo, A. Valcarce and F. Fernández, Phys. Rev. C **64**, 058201 (2001)
- **Meson spectrum**
  - J. Vijande, A. Valcarce and F. Fernández, J. Phys. G **31**, 481 (2005)
  - J. Segovia, D. R. Entem and F. Fernández, Phys. Rev. D **78** 114033 (2008)
  - J. Segovia, D. R. Entem and F. Fernández, accepted by J. Phys. G
- **Molecular states**
  - P. G. Ortega, J. Segovia, D. R. Entem and F. Fernández, Phys. Rev. D **81**, 054023 (2010)

2.- Renormalization scheme with boundary conditions

2.1.- Features

- We have two particles interacting throught a central potential

$$-u''(r) + \mathcal{U}(r)u(r) = k^2 u(r)$$

Second order differential equation $\Rightarrow$ two linear independent solutions

- One of them is the physical solution and so we have to choose the solution that is given by the regular condition at the origin

$$u(0) = 0$$

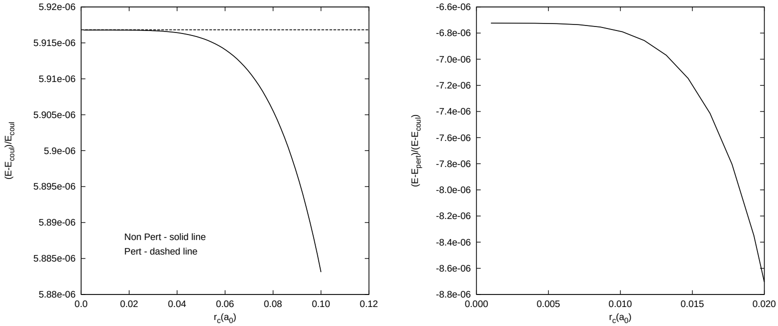
- BUT there are a great number of physical systems which we do not know the exact potential

Interaction potential is known at large distancies but not at short distancies

- Solution: We can apply boundary conditions to potentials whose short range is not known

Means that we can fix one physical observable of the large distancies and it is equivalent to impose a boundary condition at origin

2.2.- Application to hydrogen Spin-Orbit



$$V(r) = -\frac{\alpha \hbar c}{r} + \frac{\alpha (\hbar c)^3}{2m^2} \frac{1}{r^3} \vec{L} \cdot \vec{S}$$

$$n = 2 \; J = 1/2 \; L = 1 \; S = 1/2 \Rightarrow ATTRACTIVE$$

2.3.- Some recent applications and our aim

- 'Renormalization of NN-Scattering with one pion exchange and boundary conditions'
  M. Pavon Valderrama and E. Ruiz Arriola, Phys. Rev. C **70** 044006 (2004)
- 'Renormalization of the deuteron with one pion exchange'
  M. Pavon Valderrama and E. Ruiz Arriola, Phys. Rev. C **72** 054002 (2005)
- 'Renormalization and universality of Van der Waals forces'
  E. Ruiz Arriola and A. Calle Cordon, arXiv:cond-mat.other/0912.2658
- 'Low energy universality and scaling of Van der Waals forces'
  A. Calle Cordon, E. Ruiz Arriola, Phys. Rev. A 81, 044701 (2010)

OUR AIM

- *Renormalization with boundary conditions applied to our model* $\Rightarrow$ *Non perturbative treatment of our singular potential parts without cut-offs (no* $\hat{r}_0$ *and* $\hat{r}_g$*)*
- *The model in this framework has few parameters and allows us to study the correlations between physical observables and model parameters which have some physical meaning*

2.4.- Final expresions for the different model contributions

- One-gluon exchange (OGE)

$$V_{OGE}^{C}(\vec{r}_{ij}) = \frac{1}{4}\alpha_s(\vec{\lambda}_i^c \cdot \vec{\lambda}_j^c)\frac{1}{r_{ij}}$$

$$V_{OGE}^{T}(\vec{r}_{ij}) = -\frac{1}{16}\frac{\alpha_s}{m_i m_j}(\vec{\lambda}_i^c \cdot \vec{\lambda}_j^c)\frac{1}{r_{ij}^3}S_{ij}$$

$$V_{OGE}^{SO}(\vec{r}_{ij}) = -\frac{1}{16}\frac{\alpha_s}{m_i^2 m_j^2}(\vec{\lambda}_i^c \cdot \vec{\lambda}_j^c)\frac{1}{r_{ij}^3}\times$$

$$\times\left[((m_i+m_j)^2+2m_i m_j)(\vec{S}_+ \cdot \vec{L}) + (m_j^2 - m_i^2)(\vec{S}_- \cdot \vec{L})\right]$$

- Confinement

$$V_{CON}^{C}(\vec{r}_{ij}) = \left[-a_c(1-e^{-\mu_c r_{ij}}) + \Delta\right](\vec{\lambda}_i^c \cdot \vec{\lambda}_j^c)$$

$$V_{CON}^{SO}(\vec{r}_{ij}) = -\left(\vec{\lambda}_i^c \cdot \vec{\lambda}_j^c\right)\frac{a_c \mu_c e^{-\mu_c r_{ij}}}{4m_i^2 m_j^2 r_{ij}}\left[((m_i^2+m_j^2)(1-2a_s)\right.$$

$$\left.+4m_i m_j(1-a_s))(\vec{S}_+ \cdot \vec{L}) + (m_j^2 - m_i^2)(1-2a_s)(\vec{S}_- \cdot \vec{L})\right]$$

3.- Masses in charmonium from both schemes

3.1 Model parameters

Quark Mass	m_c (MeV)	1763
OGE	α_0	2.118
	Λ_0 (fm^{-1})	0.113
	μ_0 (MeV)	36.976
	$\hat{r}_0$ (fm)	0.181
	$\hat{r}_g$ (fm)	0.259
Confinement	a_c (MeV)	507.4
	μ_c (fm^{-1})	0.576
	Δ (MeV)	184.432
	a_s	0.81

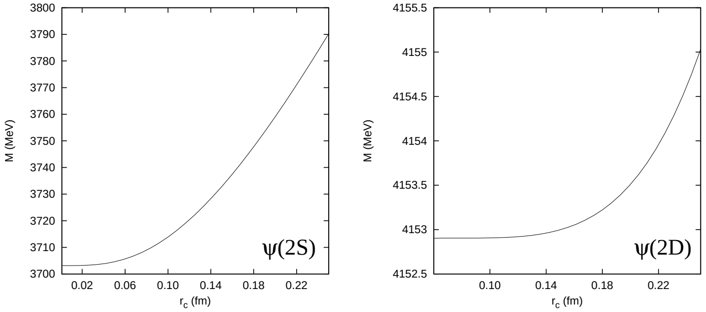
Remember: There are no cut-off in the renormalization scheme (no $\hat{r}_0$ and $\hat{r}_g$)

3.2.- Calculation

State	CQM (MeV)	$\mathcal{P}_{^3S_1}$	$\mathcal{P}_{^3D_1}$
J/ψ	3096	99.959	0.041
$\psi(2S)$	3703	99.958	0.042
$\psi(3770)$	3796	0.032	99.968
$\psi(4040)$	4097	99.935	0.065
$\psi(4160)$	4153	0.060	99.940
$\psi(4360)$	4389	99.908	0.092
$\psi(4415)$	4426	0.089	99.911
$\psi(4660)$	4614	99.884	0.116
$\psi(4660)$	4641	0.114	99.886

The mixing between S-wave and D-wave states is negligible from our original model and so we can calculate $^3S_1$ and $^3D_1$ without coupling in a renormalization scheme

3.3.- Solution stability in renormalization scheme



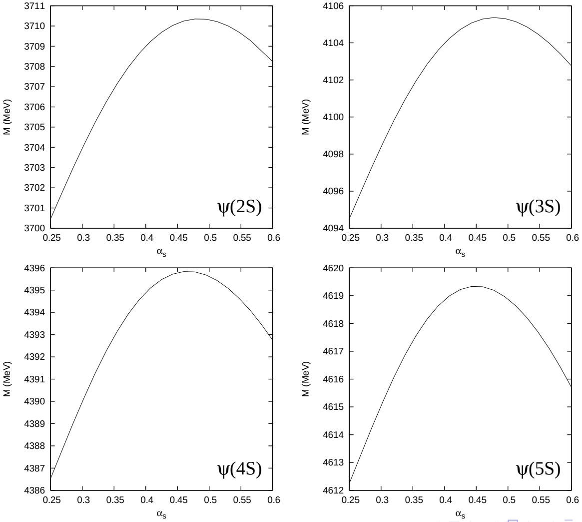
3.3.- Model results

State	n	M_{RSC} (MeV)	M_{CQM} (MeV)	M_{exp} (MeV)
$^3S_1$	1	$3096^\dagger$	3096	3096.916 ± 0.011
	2	3703	3703	3686.093 ± 0.034
	3	4097	4097	4039.6 ± 4.3
	4	4389	4389	-
	5	4614	4614	-
$^3D_1$	1	$3796^\dagger$	3796	3772.92 ± 0.35
	2	4153	4153	4153 ± 3
	3	4426	4426	4421 ± 4
	4	4641	4641	-

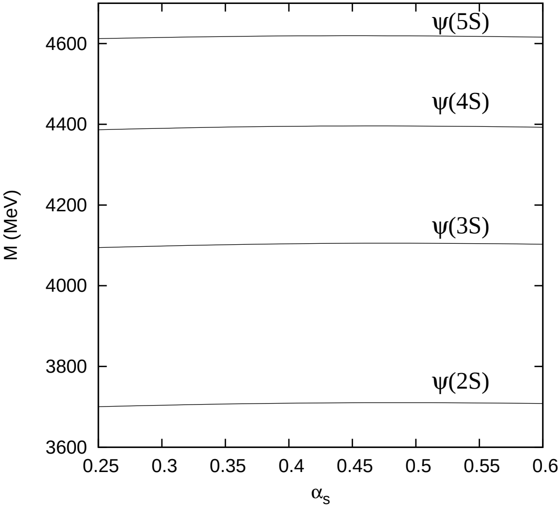
Both schemes are equivalent

4.- Study of some physical observables in function of different parameters

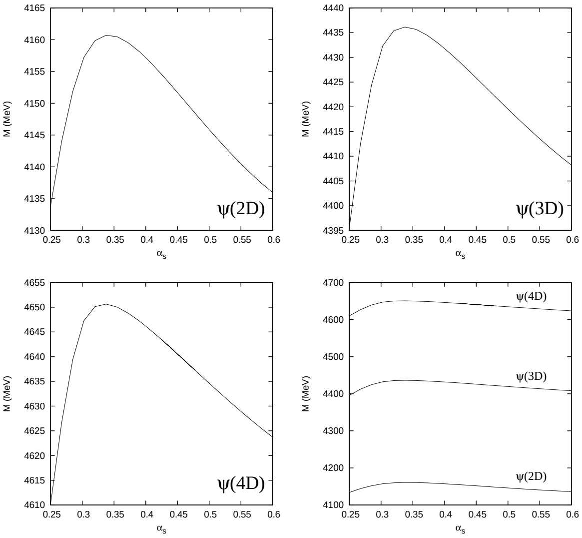
4.1.- Mases vs α_s



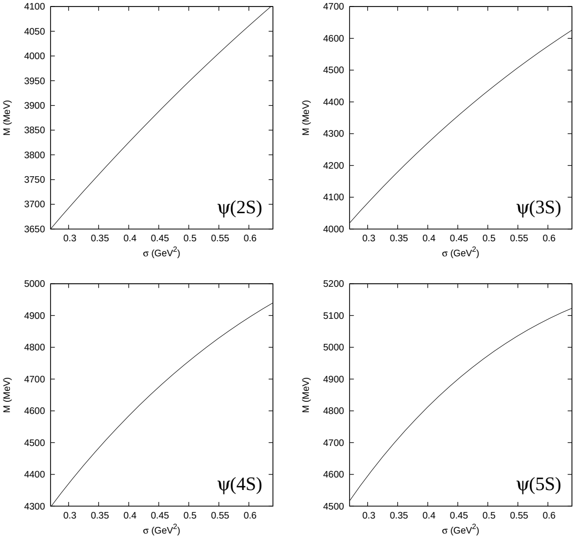
4.1.- Masses vs α_s. Continuation



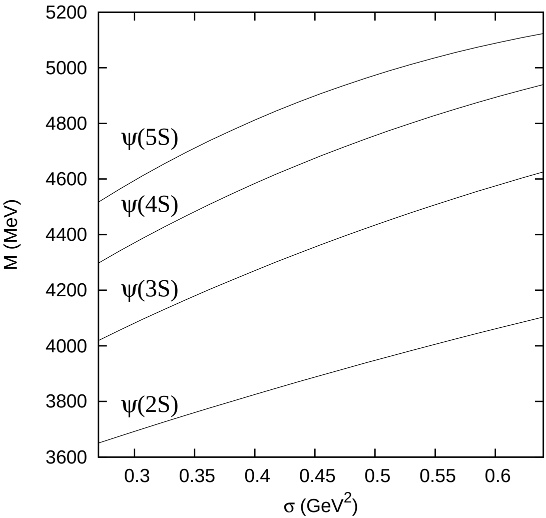
4.1.- Masses vs α_s. Continuation



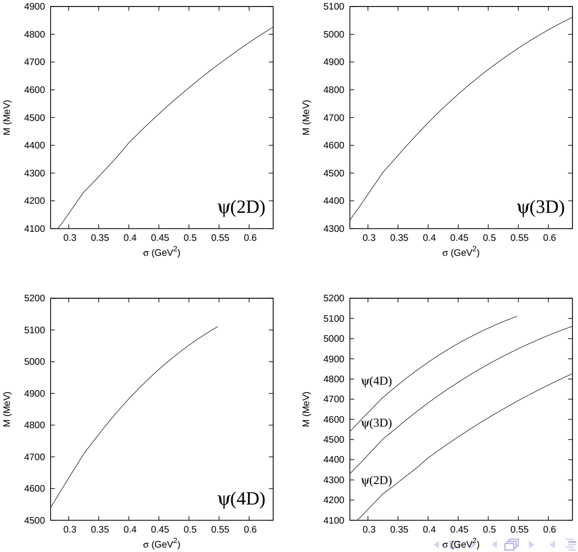
4.3.- Masses vs σ



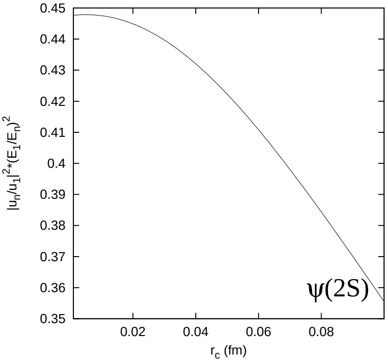
4.3.- Masses vs σ. Continuation



4.3.- Masses vs σ. Continuation



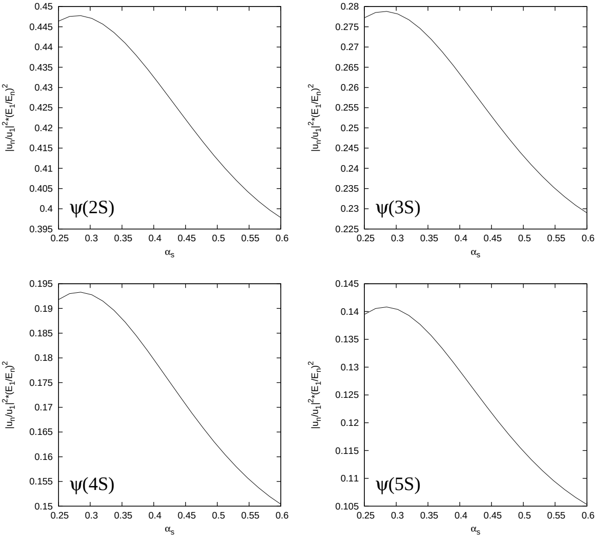
4.4.- Leptonic widths vs α_s



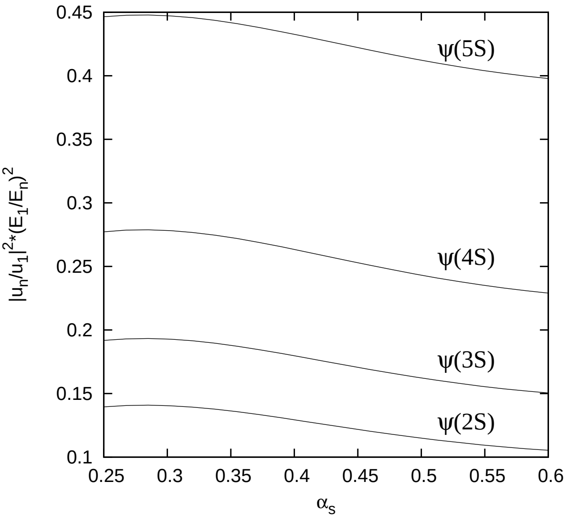
$$\Gamma(n^3S_1 \rightarrow e^+e^-) = \frac{4\alpha^2 e_c^2 |R_n(0)|^2}{E_n^2}\left(1 - \frac{16\alpha_s}{3\pi}\right)$$

$$\mathcal{R} = \frac{\Gamma(n^3S_1 \rightarrow e^+e^-)}{\Gamma(1^3S_1 \rightarrow e^+e^-)} = \frac{|R_n(0)|^2}{|R_1(0)|^2}\frac{E_1^2}{E_n^2}$$

4.4.- Leptonic widths vs α_s. Continuation



4.4.- Leptonic widths vs α_s. Continuation



5.- Conclusions

- We re-analyze the calculation of the charmonium spectrum in constituent quark model using a renormalization boudary condition scheme
- We find a good agreement between both schemes which provides confidence on the way the original model take into account the unknown short distance dynamics
- The use of this scheme allows us to further study the dependence of the states on the model parameters in a cleaner way since the regulator dependence has been removed when a suitable renormalization condition is imposed
- We find:
  - The mass of the excited states strongly depend on the string tension parameter
  - There is a remarkable insensitivity to the strong coupling constant entering the one gluon exchange contribution to the potential. This avoids a great deal of unphysical fine tuning which suggested taking for this parameter unnaturally large values $\alpha_s \sim 0.3 - 0.4$
  - The leptonic widths depend strongly on the strong coupling constant. As expected because is a short range observable