J/ψ as an ideal system to study high order QCD corrections

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Contents

1.- Context of $J/\psi \longrightarrow 3\gamma$ decay

1.1- Experimental data

- Phys. Rev. Lett. **101**, 101801 (2008)
    - Cleo Collaboration $\rightarrow \mathcal{B}(J/\psi \rightarrow 3\gamma)$
    - $\psi(2S) \rightarrow \pi^+\pi^- J/\psi$ events
    - CLEO-C detector operating at the CESR e^+e^- collider
    - The signal has a statistical significance of 6σ and corresponds to $\mathcal{B}_{3\gamma} = (1.2 \pm 0.3 \pm 0.2) \times 10^{-5}$
- Ignoring QCD corrections

$$\frac{B_{3\gamma}}{B_{e^+e^-}} \sim \frac{\alpha}{14}$$

$$B_{e^+e^-} = (5.94 \pm 0.06) \times 10^{-2}\,(PDG)$$

$$B_{3\gamma} \sim 3 \times 10^{-5}$$

1.2.- Naive estimation

- Ortho-positronium (o-Ps) is a bound state of e^+e^- with quantum numbers ${}^3S_1$ and it decays to 3γ almost exclusively:

$$\Gamma_0 = \frac{16}{9}\frac{(\pi^2-9)}{m^2}\alpha^3 e^6|\varphi(0)|^2$$

- The analog to o-Ps $\rightarrow 3\gamma$ for QCD is the three-photon vector quarkonium decay $J/\psi \rightarrow 3\gamma$:

$$\Gamma_0(J/\psi \rightarrow 3\gamma) = \frac{4(\pi^2-9)\alpha^3 e_c^6}{3\pi m_c^2}|R_{10}(0)|^2$$

- We must use the Van-Royen equation for the $J/\psi \rightarrow e^+e^-$ decay

$$\Gamma(1\,{}^3S_1 \rightarrow e^+e^-) = \frac{4\alpha^2 e_c^2|R_{10}(0)|^2}{(2m_c)^2}\left[1 - \frac{16\alpha_s}{3\pi} + \Delta(n\,{}^3S_1)\right]$$

2.- Constituent quark model and its results

2.1.- Most important ingredients / J. Phys. G: Nucl. Part. Phys. **31**, 481 (2005)

- Spontaneous chiral symmetry breaking (Goldstone-Bosons exchange):

$$L = \bar{\psi}\left(i\gamma^{\mu}\partial_{\mu} - MU^{\gamma_5}\right)\psi \rightarrow \quad U^{\gamma_5} = 1 + \frac{i}{f_\pi}\gamma^5\lambda^a\pi^a - \frac{1}{2f_\pi^2}\pi^a\pi^a + \ldots$$

$$M(q^2) = m_q F\left(q^2\right) = m_q\left[\frac{\Lambda^2}{\Lambda^2+q^2}\right]^{1/2}$$

- QCD perturbative effects (One gluon exchange):

$$L = i\sqrt{4\pi\alpha_s}\,\bar{\psi}\gamma_\mu G^\mu\lambda^c\psi$$

- Confinement (screened potential):

$$V_{CON}^{C}(\vec{r}_{ij}) = \left[-a_c(1-e^{-\mu_c r_{ij}}) + \Delta\right](\vec{\lambda}_i^c \cdot \vec{\lambda}_j^c)$$

$$\begin{cases} V_{CON}^{C}(\vec{r}_{ij}) = (-a_c\mu_c r_{ij} + \Delta)\,(\vec{\lambda}_i^c \cdot \vec{\lambda}_j^c) & r_{ij} \rightarrow 0 \\ V_{CON}^{C}(\vec{r}_{ij}) = (-a_c + \Delta)(\vec{\lambda}_i^c \cdot \vec{\lambda}_j^c) & r_{ij} \rightarrow \infty \end{cases}$$

2.2.- Some applications

- **N-N interaction**
  - F. Fernandez, A. Valcarce, U. Straub and A. Faessler, J. Phys. **G19**, 2013 (1993)
  - A. Valcarce, A. Faessler and F. Fernandez, Phys. Lett. **345**, 367 (1995)
  - D. R. Entem, F. Fernandez and A. Valcarce, Phys. Rev. **C62**, 034002 (2000)
  - B. Julia-Diaz, J. Haidenbauer, A. Valcarce and F. Fernandez, Phys. Rev. **C65**, 034001 (2002)
- **Baryon spectrum**
  - H. Garcilazo, A. Valcarce and F. Fernandez, Phys. Rev. **C63**, 035207 (2001)
  - H. Garcilazo, A. Valcarce and F. Fernandez, Phys. Rev. **C64**, 058201 (2001)
- **Meson spectrum**
  - L. A. Blanco, F. Fernandez and A. Valcarce, Phys. Rev **C59**, 428 (1999)
  - J. Vijande, A. Valcarce and F. Fernandez, J. Phys. **G31**, 481 (2005)

2.3.- $J^{PC} = 1^{--}$ $c\bar{c}$ mesons
Phys. Rev. **D78**, 114033 (2008)

(nL)	States	QM	Exp	Ref.
(1S)	J/ψ	3096	3096.916 ± 0.011	[1]
(2S)	$\psi(2S)$	3703	3686.093 ± 0.034	[1]
(1D)	$\psi(3770)$	3796	3775.2 ± 1.7	[1]
	$Y(4008)$		4008 ± 40	[2]
(3S)	$\psi(4040)$	4097	4039 ± 1	[1]
(2D)	$\psi(4160)$	4153	4153 ± 3	[1]
	$Y(4260)$		4260 ± 10	[3]
(4S)	$\psi(4360)$	4389	4361 ± 9	[4]
(3D)	$\psi(4415)$	4426	4421 ± 4	[1]
$\begin{bmatrix}(5S)\\(4D)\end{bmatrix}$	$\psi(4660)$	$\begin{bmatrix}4614\\4641\end{bmatrix}$	4664 ± 11	[4]

[1] Phys. Lett. **B667**, 1 (2008)
[2] Phys. Rev. Lett. **99**, 182004 (2007)
[3] Phys. Rev. Lett. **95**, 142001 (2005)
[4] Phys. Rev. Lett. **99**, 142002 (2007)

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2.4.- Leptonic Widths
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(nL)	states	QM (MeV)	Γ_{QM} (keV)	Γ_{exp} (keV)	Ref.
(1S)	J/ψ	3096	3.93	5.55 ± 0.14	[1]
(2S)	$\psi(2S)$	3703	1.78	2.33 ± 0.07	[1]
(1D)	$\psi(3770)$	3796	0.22	0.22 ± 0.05	[2]
(3S)	$\psi(4040)$	4097	1.11	0.83 ± 0.20	[2]
(2D)	$\psi(4160)$	4153	0.30	0.48 ± 0.22	[2]
(4S)	$\psi(4360)$	4389	0.78		
(3D)	$\psi(4415)$	4426	0.33	0.35 ± 0.12	[2]
$\begin{bmatrix}(5S)\\(4D)\end{bmatrix}$	$\psi(4660)$	$\begin{bmatrix}4614\\4641\end{bmatrix}$	$\begin{bmatrix}0.57\\0.31\end{bmatrix}$		

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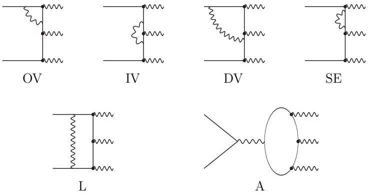
3.- QCD corrections to $J/\psi \longrightarrow 3\gamma$ decay

3.1.- What happens in QED?

- Expression at order zero:

$$\Gamma_0 = \frac{16}{9}\frac{(\pi^2-9)}{m^2}\alpha^3 e^6 |\varphi(0)|^2$$

- One loop diagrams that contribute to the decay amplitude at first order:



- Expression at first order (Phys. Rev. Lett. **84**, 5086 (2000)):

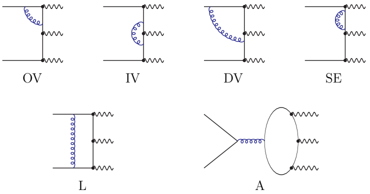
$$\Gamma = \Gamma_0 \left[1 + A_{QED}\left(\frac{\alpha}{\pi}\right)\right] \text{ with } A_{QED} = -10.286606$$

3.2.- One loop QCD corrections

- Expression at order zero:

$$\Gamma_0(J/\psi \to 3\gamma) = \frac{4(\pi^2-9)\alpha^3 e_c^6}{3\pi m_c^2}|R_{10}(0)|^2$$

- One loop diagrams that contribute to the decay amplitude at first order ($\lambda_c/2$ in each gluon vertex which contributes with a global factor 4/3):



- Expression at first order:

$$\Gamma = \Gamma_0\left[1 + A_{QCD}\left(\frac{\alpha_s}{\pi}\right)\right] \text{ with } A_{QCD} = -12.630$$

3.2.- One loop QCD corrections. Continuation

- Using the value of the $\alpha_s(Q^2)$ coupling constant at the charm quark mass scale ($\alpha_s = 0.288$):

$$\Gamma = 3.4 \times 10^{-5} \left[1 - 12.630 \left(\frac{0.288}{\pi}\right)\right] \sim 0$$

> $\Rightarrow$ *The first correction to the width cancels with the zeroth order approximation given a theoretical prediction compatible with zero*

- Because of the strong cancellation between the first two terms
  $\longrightarrow$ the most important contribution comes from the $\left(\frac{\alpha_s}{\pi}\right)^2$

3.3.- At second order in α_s

- This correction has two different sources
  - The square of one loop corrections to the annihilation amplitude
  - The leading order two loop corrections
- We can naively estimate the first contribution as:

$$B_{1,QCD} = \left(\frac{A_{QCD}}{2}\right)^2$$

- The final result is:

$$\Gamma(J/\psi \to 3\gamma) = 3.17\times 10^{-3}\left[1 - 12.630\left(\frac{\alpha_s}{\pi}\right) + 39.879\left(\frac{\alpha_s}{\pi}\right)^2\right]\ \text{keV}$$

$$B_{3\gamma} = 0.6 \times 10^{-5}$$
$$\Downarrow$$
Lower than the experimental data but near the experimental error bar

4.- Conclusions

- $J/\psi \rightarrow 3\gamma$ decay is an ideal observable to study high order QCD corrections
- The naive estimation of the second order corrections give a result that is in reasonable agreement with the recent reported experimental data
- Although the naive calculation is in good agreement with experimental data, this value is lower and then the two loop corrections must be important